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An Inventory Model with Commercial Efforts for Deteriorating Items under Fuzzy and Learning Concept

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ABSTRACT

The present paper deals with fuzzy based inventory model with advertisement efforts for deteriorating items under learning effect. The demand for items is a more effective tool for any business sector or any firm, as well as various types of flesh factories, and also affects the total inventory cost or profit of the inventory system. In this paper, we assumed the demand rate depends on the stock and advertisement efforts and holding cost per unit item follows the effect of imprecise in nature. The holding cost per unit is modeled as a triangular fuzzy number, and the ordering cost is affected by learning. The effect of fuzziness, learning and advertisement effect got positive on the total inventory cost or the profit. The numerical example has been presented for the justification of the proposed model. The sensitivity analysis has also been shown for the decision maker for the application of the present model.

Keywords: EOQ, Fuzzy environment, Learning effect, Greening efforts, Advertisement effect.

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1. INTRODUCTION

The advertisement is the best policy which increases the selling of the items and advertisement improves the demand rate of the items. A lot of companies, firm and industries are used advertisement efforts for more selling and included some advertisement cost for advertising of items. The learning is a mathematical tool which constructs the order quantity for the ordering policies in the business market.

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For the development of this proposed model, we studied a lot of literature review and also included some selected literature review in the further paragraph.

A model incorporating quantity discounts, time-dependent storage costs, and selling price-dependent demand was presented by Ghaithan and Alfares (2016). A model of inventory for degrading stock and demand was developed by Nita et al. (2022) and is based on stock, selling price, green efforts, and a holding cost that varies with time. A finite horizon EOQ model with a partial backlog and demand that is influenced by price and advertisement impact was developed by Palanivel et al. (2015). Taleizadeh et al. (2020) took partial backordering, carbon emissions, and all unit quantity discounts into account in their inventory model. An inventory model with holding costs based on time, shortages, and reimbursement delays was covered by Saha and Sen (2018).Masanta and Giri are creating a supply chain model that depends on demand, selling price, and quantity (2020).A decaying inventory model was developed by Weber et al. (2020) under preservation technology, with demand determined by selling price and stock size. A time-dependent carrying cost model for the greening degrading products under a greening environment, where demand fluctuates with selling price and greening factor, was suggested by Paul et al. in 2022. According to this perspective, a preservation-based model with leaning for perishable goods under credit policy was given by Jayaswal et al. (2021).

Presumably, the vendor exchanges a new style of product with a short lifespan, such as an updated film or fashionable apparel. Typically, before a seller publishes how many things they have for sale, the market's demand for the newest products is erratic and difficult to predict with precision. Decision-makers may be poisoned by the uncertainty in the order of the goods, thus it is crucial to handle the ambiguity in the demand order carefully. Many inventory models treat demand, which is recognised under many distributions, as a random variable in the supply chain system.But given the volume of mathematical interpretations needed to meet the requirements, a number of writers recognised that it isn't practical for everyone to express the uncertainty in a random manner.

In supply chain inventory models, where certain inventory parameters were uncertain in nature, Zadeh (1965) first reported the use of fuzzy concepts to resolve various kinds of uncertain problems. These models also took uncertain order into account when defining fuzzy parameters, and they were then defuzzified using defuzzification techniques to solve the problems at hand. Using trapezoidal fuzzy parameters, Park (1987) created an inventory model for the ideal order quantity while minimising the cost of inventory. Lam and Wong (1996) expanded Dolan's (1978) fuzzy-based work for the deterministic model to include commercial implications. An inventory mathematical model for economic order quantity was presented by Yao and Lee (1996), in which the order quantity was treated as a fuzzy number under the permitted shortages.

By treating the ordering and carrying costs as fuzzy numbers, Vujasevic et al. (1996) enhanced a mathematical model for a single player and optimised the order quantity. An EOQ model with backorder was studied by Chen et al. (1996) using the fuzzy idea. Mandal et al. (1998) summarised a fuzzy inventory model for deteriorating items where demand is a function of supply and limited storage space.Roy and Maiti (1998) developed a multi-objective mathematical representation for the decaying items under the fuzzy notion with some inventory characteristics. Yao and Su (2000) focused on various situations with and without backorders under fuzzy production-inventory models. In Hsieh's 2002 study, inventory models were introduced in a fuzzy environment. Certain inventory parameters were treated as fuzzy numbers in order to determine the maximum EPQ using recently developed techniques, including the Extended Lagrangean (EL) approach and the Graded Mean Integration (GMI) method. A buyer-seller inventory model for deteriorating products was developed by Yang and Wee (2000) using the heuristic approach theory. Liu (2000) introduced a new basis for the stock model in optimisation theory, which treats imprecise values as fuzzy parameters. Skouri and Papachristos (2002) adopted a lot-sizing mathematical inventory model for a decaying system, where demand is not stable under inflation and shortages occur.

In a fuzzy environment, where the whole demand and carrying cost are considered as fuzzy numbers with or without the backorders method, Yao and Chiang (2003) included some new strategies and established a mathematical inventory representation for the economic order quantity and generated a new total cost function. De et al. (2003) enhanced an inventory formulation for the EOQ model, in which the fading rate and demand rate function as fuzzy variables. Das et al. (2004) examined a fuzzy mathematical stock formulation for buyers and retailers about decaying goods under rebate policies. Mahata et al. (2005) proposed several novel approaches for determining the lot size for buyer and seller in a murky business environment. Numerous writers worked on this subject after it. In an EOQ model where inflation and fading rates are seen as fuzzy variables under the credit financing policy, De and Goswami (2006) solved this issue. Eric Sucky (2006) explained the two-level supply chain model in terms of the buyer-seller coordination method. Mahata and Goswami (2007) developed an EOQ inventory formulation for deteriorating products with credit financing strategy under fuzzy notion. Sinha and Sarmah (2008) developed a mathematical solution for effective seller-buyer coordination in a supply chain with fuzzy inventory characteristics. By assuming that different inventory parameters are fuzzy numbers, Gani and Maheswari (2010) suggested an inventory model for EOQ in a fuzzy environment. Jaggi et al. (2012) provided an explanation of the EOQ mathematical representation of stock for decomposing items under the credit financing strategy and shortages, taking into account the impact of inspection in fuzzy systems. The EOQ model for decomposing objects in a fuzzy environment with time-dependent demand and shortages was covered by Jaggi et al. (2012). Jaggi and Sharma (2014) used a fuzzy environment to build an economic order quantity taking shortages and credit financing policies into account. According to the fuzzy notion, Sharma et al. (2017) suggested the best refill technique for cost-reliant orders in various economic scenarios. Patro et al. (2018) presented a fuzzy environment for decomposing objects with the goal of teaching users how to learn from items with flawed features through equitable concession.

2. ASSUMPTION AND NOTATION

$\tilde{h}$ Fuzzy holding cost of the items (in dollars/unit);

(h_1, h_2, h_3)Triangular fuzzy number

q_j The maximum quantity that must be ordered at costP_j, where $j = 1, \ldots, J$

P_j **The cost for** Purchase unit for quantity q of orders,(in dollars/unit) when $q_{j-1} \leq q \leq q_j$

s The cost for good sold unit (in dollars/unit);

C_O Ordering cost(in dollars/order)

C_A Per-item cost of advertising (in dollars/unit);

g Fixed part of the holding cost (in dollars/unit);

h Holding cost of the items (in dollars/unit);

α Shape of the demand rate; > 0;

β Stock-dependent parameter $\beta > 0$, $\beta < \alpha$;

ψ Environmentally sensitive parameter, $0 \leq \psi < 1$;

δ Advertising elasticity of the demand function where $0 \leq \delta < 1$;

γ Cost-sensitive greening parameter, $\gamma > 0$;

μ Selling parameters for the items, $\mu > 1$;

η the effort for greening,$\eta > 0$;

$I(t)$ the inventory level at t; $0 \leq t \leq T$;

$D(I(t), s, \eta, C_A)$The rate of demand, which depends on the stock, the price for the selling items, the greening effort, and the advertisement cost.

θ**:** Fixed decaying rate with $0 \leq \theta < 1$;

T**:** Whole cycle length (in years) (decision variable)

s The price for the selling items (in units); (decision variable)

η The greening effort for the items

TP Whole profit for the buyer (in dollars/years);

$\widetilde{TP}$ Whole fuzzy profit for the buyer (in dollars/years);

2.1 Model's assumption

- This model is concentrated on a single item.
- In this model no lead time is allowed.
- Instantaneous replenishment is possible.
- The unit's overall charge is presumptive.
- As a falling function of lot size Q,

The purchasing cost for the items can be defined for this model:

$$C(Q) = \boldsymbol{P_j},$$

$$if\ q_{j-1} \leq Q < q_j,$$

Purchase cost Discount level

P_0	$0 \leq Q < q_1$
P_1	$q_1 \leq Q < q_2$
P_2	$q_2 \leq Q < q_3$
$.P_j$	$Q \geq q_j$

The rate of demand can be finding out by the calculating of stock level, selling price, greening effort and advertisement cost and it can be write

$$\boldsymbol{D(I(t), s, \eta, C_A) = {C_A}^{\delta}(\alpha + \beta I(t) + \psi\eta)s^{-\mu}}$$

- The deterioration rate is fixed.
- Holding cost per unit item has been treated as triangular fuzzy number.
- Holding Cost is calculated as a function of time and the price of acquisition and is defined as

$$H(t) = (g + ht).P_j$$

Where, $g =$ The fixed component and the variable component,$h$of the carrying cost and treated as a triangular fuzzy number.

- The total fuzzy profit defuzzified with the help of the centroid method.
- The learning effect is involved in the ordering cost.

3. MATHEMATICAL FORMULATION UNDER CRISP ENVIRONMENT

Suppose that at initial when $t = 0$, then the order lot size is Q units. The level of the stock is reducing due to demand and deterioration and this inventory level will be zero at T. Let us consider that $I(t)$ be a inventory level at time $t$which follows the differential equation with the boundary condition$I(T) = 0$. The differential equation can be represented below

$$\frac{dI(t)}{dt} + \theta I(t) = -{C_A}^{\delta}(\alpha + \beta I(t) + \psi\eta)s^{-\mu} \ldots\mathbf{(1)}$$

After resolving the first order differential equation above, we obtain,

$I(t)=\frac{C_A{}^{\delta}(\alpha+\psi\eta)s^{-\mu}}{\theta+C_A{}^{\delta}\beta s^{-\mu}}(e^{(\theta+C_A{}^{\delta}\beta s^{-\mu})(T-t)}-1)$ **….(2)**

By using the boundary condition, $I(0)=Q$, we get

$Q=\frac{C_A{}^{\delta}(\alpha+\psi\,\eta)s^{-\mu}}{\theta+C_A{}^{\delta}\beta s^{-\mu}}(e^{(\theta+C_A{}^{\delta}\beta s^{-\mu})T}-1)$….**(3)**

Now we will define different costs:

i. Set up cost $(OC)=\frac{\mathbf{1}}{\boldsymbol{T}}\left(\boldsymbol{C_1}+\frac{\boldsymbol{C_2}}{\boldsymbol{n^{\beta}}}\right)$**…. (4)**

ii. Purchasing Cost $(PC)=\frac{P_j.Q}{T}$….(5)

iii. Holding Cost (HC) =

$$\frac{P_j}{T}\int_0^T(g+ht)I(t)dt$$

$$=\frac{C_A{}^{\delta}P_j(\alpha+\psi\eta)}{T(\beta C_A{}^{\delta}+\theta s^{\mu})}\left(-gT-h\frac{T^2}{2}+\frac{e^{T(\theta+\beta C_A{}^{\delta}s^{-\mu})}s^{\mu}((h+g\theta)s^{\mu}+gC_A{}^{\delta}\beta)}{(C_A{}^{\delta}\beta+\theta p^{n})^2}+\frac{s^{\mu}(-C_A{}^{\delta}(g+hT)\beta-s^{\mu}(h+g\theta+hT\theta))}{(C_A{}^{\delta}\beta+\theta s^{\mu})^2}\right)\text{…..(6)}$$

iv. Greening Effort Investment (GI)=

$$\frac{1}{T}\int_0^T\int_0^{\eta}(\gamma\eta)d\eta dt$$

$$=\frac{1}{2}\gamma\eta^2\text{…(7)}$$

Now, we calculate the total income from 0 to T

Total sales income $(SR)=\frac{1}{T}\int_0^T\left((\alpha+\beta I(t)+\psi\eta)s^{-\boldsymbol{\mu}+1}\right)dt$

$$=\frac{s(s^{\mu}T\theta^2+C_A{}^{\delta}\beta(-1+T\theta+e^{T(\theta+\beta C_A{}^{\delta}s^{-\eta})})(\alpha+\psi\eta)}{T(C_A{}^{\delta}\beta+\theta s^{\mu})^2}\text{…}\mathbf{(8)}$$

The total Profit function per unit time is

$$\varphi=\frac{(SR-(OC+PC+HC+GI))}{T}$$

$$\varphi = \frac{s(s^{\mu}T\theta^{2}+C_{A}{}^{\delta}\beta(-1+T\theta+e^{T\left(\theta+\beta C_{A}{}^{\delta}s^{-\eta}\right)})(\alpha+\psi\eta)}{T\left(C_{A}{}^{\delta}\beta+\theta s^{\mu}\right)^{2}} - \frac{\left(\boldsymbol{C_{1}}+\frac{\boldsymbol{C_{2}}}{\boldsymbol{n\beta}}\right)}{\boldsymbol{T}} + \frac{P_{j}.Q}{T} + \frac{C_{A}{}^{\delta}P_{j}(\alpha+\psi\eta)}{T(\beta C_{A}{}^{\delta}+\theta s^{\mu})}\left(-gT - h\frac{T^{2}}{2} + \frac{e^{T\left(\theta+\beta C_{A}{}^{\delta}s^{-\mu}\right)}s^{\mu}((h+g\theta)s^{\mu}+gC_{A}{}^{\delta}\beta)}{\left(C_{A}{}^{\delta}\beta+\theta p^{n}\right)^{2}} + \frac{s^{\mu}\left(-C_{A}{}^{\delta}(g+hT)\beta-s^{\mu}(h+g\theta+hT\theta)\right)}{\left(C_{A}{}^{\delta}\beta+\theta s^{\mu}\right)^{2}}\right) + \frac{1}{2}\gamma\eta^{2}) \quad \ldots\textbf{(9)}$$

4. Mathematical formulation under fuzzy environment

In this paper, holding cost per unit item treated as triangular fuzzy number which is denoted by $(\boldsymbol{h_{1}},\boldsymbol{h_{2}},\boldsymbol{h_{3}})$and defuzzifiedwith the help of centroid method. From the equation (9), we converted in to fuzzy environment.

The total fuzzy profit per unit time

$$\tilde{\varphi} = \frac{s(s^{\mu}T\theta^{2}+C_{A}{}^{\delta}\beta(-1+T\theta+e^{T\left(\theta+\beta C_{A}{}^{\delta}s^{-\eta}\right)})(\alpha+\psi\eta)}{T\left(C_{A}{}^{\delta}\beta+\theta s^{\mu}\right)^{2}} - \frac{\left(\boldsymbol{C_{1}}+\frac{\boldsymbol{C_{2}}}{\boldsymbol{n\beta}}\right)}{\boldsymbol{T}} + \frac{P_{j}.Q}{T} + \frac{C_{A}{}^{\delta}P_{j}(\alpha+\psi\eta)}{T(\beta C_{A}{}^{\delta}+\theta s^{\mu})}\left(-gT - \tilde{h}\frac{T^{2}}{2} + \frac{e^{T\left(\theta+\beta C_{A}{}^{\delta}s^{-\mu}\right)}s^{\mu}((\tilde{h}+g\theta)s^{\mu}+gC_{A}{}^{\delta}\beta)}{\left(C_{A}{}^{\delta}\beta+\theta p^{n}\right)^{2}} + \frac{s^{\mu}\left(-C_{A}{}^{\delta}(g+\tilde{h}T)\beta-s^{\mu}(\tilde{h}+g\theta+\tilde{h}T\theta)\right)}{\left(C_{A}{}^{\delta}\beta+\theta s^{\mu}\right)^{2}}\right) + \frac{1}{2}\gamma\eta^{2})$$

(10)

The total fuzzy profit defuzzified with the help of centroid method, we get

$$\tilde{\tilde{\varphi}} = \frac{s(s^{\mu}T\theta^{2}+C_{A}{}^{\delta}\beta(-1+T\theta+e^{T\left(\theta+\beta C_{A}{}^{\delta}s^{-\eta}\right)})(\alpha+\psi\eta)}{T\left(C_{A}{}^{\delta}\beta+\theta s^{\mu}\right)^{2}} - \frac{\left(\boldsymbol{C_{1}}+\frac{\boldsymbol{C_{2}}}{\boldsymbol{n\beta}}\right)}{\boldsymbol{T}} + \frac{P_{j}.Q}{T} + \frac{C_{A}{}^{\delta}P_{j}(\alpha+\psi\eta)}{T(\beta C_{A}{}^{\delta}+\theta s^{\mu})}\left(-gT - \left(\frac{(h_{1}+h_{2}+h_{3})}{3}\right)\frac{T^{2}}{2} + \frac{e^{T\left(\theta+\beta C_{A}{}^{\delta}s^{-\mu}\right)}s^{\mu}(\left(\left(\frac{(h_{1}+h_{2}+h_{3})}{3}\right)+g\theta\right)s^{\mu}+gC_{A}{}^{\delta}\beta)}{\left(C_{A}{}^{\delta}\beta+\theta p^{n}\right)^{2}} + \frac{s^{\mu}\left(-C_{A}{}^{\delta}\left(g+\left(\frac{(h_{1}+h_{2}+h_{3})}{3}\right)T\right)\beta-s^{\mu}\left(\left(\frac{(h_{1}+h_{2}+h_{3})}{3}\right)+g\theta+\left(\frac{(h_{1}+h_{2}+h_{3})}{3}\right)T\theta\right)\right)}{\left(C_{A}{}^{\delta}\beta+\theta s^{\mu}\right)^{2}}\right) + \frac{1}{2}\gamma\eta^{2})$$

(11)

5. Algorithm for the calculation of total fuzzy profit

We followed the solving algorithm according the Nita et al. (2022):

Step1: First, set the total fuzzy profit function, $\tilde{\tilde{\varphi}}*$, to zero. and $j = J$

Step2: Apply all the mathematical appropriate units,

$n, \alpha,\ \beta,\ (\boldsymbol{h_1}, \boldsymbol{h_2}, \boldsymbol{h_3}),\ g,\ C_1, C_2, l, \gamma\ ,\ \mu,\ \theta,\ \psi, C_A\ , \delta$,and

P_j.

Step3: Calculate Pj's attainable criterion as follows:

Consider decision-making factors T, s, η by substituting

$$\frac{\partial\tilde{\tilde{\varphi}}}{\partial s} = 0, \frac{\partial\tilde{\tilde{\varphi}}}{\partial T} = 0 and\ \frac{\partial\tilde{\tilde{\varphi}}}{\partial \eta} = 0\ (12)$$

Step 4: Check if $q_j \leq Q < q_{j+1}$then the answer is reachable. Replacement values of T, s, η in equation 4 to get TP_j

TP_j should be used in place of TP* if $TP_j >$ TP*.

If so, proceed to step 7; otherwise, go to step 5.

Step 5: Assuming P_j as the value now, Q =q_j and if solved the equation number (5)and also calculates the decision variables s, T, andηafter that we calculate the profit of this system, $\tilde{\tilde{\varphi}}_j$ are obtained.

If $\tilde{\tilde{\varphi}}_j > \tilde{\tilde{\varphi}}^*$, then we will consider, $\tilde{\tilde{\varphi}}^* = \tilde{\tilde{\varphi}}_j$.

After that we go to the step 6.

Step 6: If $j \geq 2$, we will consider$j = j - 1$.

Now go to the step 1.

Step 7: After that, we calculate the decision variables T, s and η with Q and also find out the optimal profit $\tilde{\tilde{\varphi}}*$ with respect to decision variable.

6. EXAMPLE

Consider the following:

$\alpha = 1600, \beta = 5, g = 0.8$\$/unit/year,$(\boldsymbol{h_1}, \boldsymbol{h_2}, \boldsymbol{h_3}) = (\mathbf{0.14}, \mathbf{0.15}, \mathbf{0.16})$,$C_1 =$ 200\$/order,$C_2 = 100$ \$/order, $n = 5, l = 0.79, \psi = 2, , \eta = 1.1, \theta = 0.2, \gamma = 2, \delta = 0.3, P_1 =$ 3.5\$/unit, $P_2 = 3.25$\$/unit, $P_3 = 3$\$/unit, $q_1 = 0$units, $q_2 = 90$units, $q_3 = 180$ units, $C_A =$ 0.9\$/unit.

Solution using the above algorithm:

Step 1: Take $\tilde{\tilde{\varphi}}^* = 0,\ j = 3$

$$(Q \geq 180)$$

Step 2: Put mathematical values to the inventory parameters.

Step 3: For $P_3 = 3$**,** the optimal measure is computed, Software called MATHEMATICA 12.0 is used.,we get, $T = 5.553$ years**,** $\eta = 0.78$**,** $s = 35.28$\$/unit

And hence $Q = 178$ units.

And $\tilde{\tilde{\varphi}}. = 1151.058$ \$/year

Here, $Q < 180$.

Step 4: Here, $Q < 180$, so, problem cannot be solved, Move tostep 5.

Step 5: Assume Q is 180 and P_3 is the cost price at this point. The values are then obtained by using MATHEMATICA 12.0.

$T = 1.799$ years, $\eta = 0.591$,

$s = 689.78$ \$/unit

And $\tilde{\tilde{\varphi}} = 809.78$ \$/year.

Selling price is not attainable here. Proceed to step 6.

Step 6: Now take$j = 2$ and follow step 1. For $P_2 = 3.25, (90 \leq Q < 180)$

We get the values,$T = 5.553$ years.

$\eta = 0.78, s = 35.28\$$ \$/unit.

And $TP = 926.48$ \$/year.

And hence $Q = 178$ units.

Step 7: Therefore, the optimal solution, is $TP = 1151.058\$$/years.

7. SENSITIVITY ANALYSIS

In this section, we analyzed the effect of learning, shipment, deterioration rate, and fuzzy number, which are shown in Table 1.

Table 1: The observations for the sensitivity analysis function

Parameters	Value of parameters	T	η	s	Q	$\tilde{\tilde{\varphi}}^*$
n	1	2.123	0.79	25.24	200	1389
	2	3.324	0.79	26.24	190	1369
	3	4.567	0.79	27.24	188	1255
	5	5.553	0.78	35.28	178	1151
θ	0.16	5.983	0.98	30.78	201	1345
	0.18	5.765	0.84	32.07	190	1250
	0.2	5.553	0.78	35.28	178	1151
	0.22	5.456	0.65	38.68	150	1098
α	1280	5.890	0.82	38.71	150	891
	1440	5.678	0.79	37.34	167	927
	1600	5.553	0.78	35.28	178	1151
	1760	5.534	0.72	34.33	126	1231

$\boldsymbol{\beta}$	4	3.9	0.71	48.17	110	1031
	4.5	4.1	0.75	41.93	150	1104
	5	5.5	0.78	35.28	178	1151
	5.5	6.5	0.82	32.89	161	1165
$\boldsymbol{l}$	0.71	5.5	0.78	35.28	121	934.19
	0.81	5.5	0.78	35.28	119	930.11
	0.79	5.5	0.78	35.28	178	1151
	0.99	5.5	0.78	35.28	178	1151
$(\boldsymbol{h_1, h_2, h_3})$	$(\mathbf{0.12, 0.13, 0.14})$	3.3	0.71	23	190	1453
	$(\mathbf{0.14, 0.15, 0.16})$	4.4	0.72	38	187	1389
	$(\mathbf{0.17, 0.18, 0.19})$	5.5	0.78	35.28	178	1151
	$(\mathbf{0.19, 0.20, 0.21})$	5.9	0.79	45	118	1091

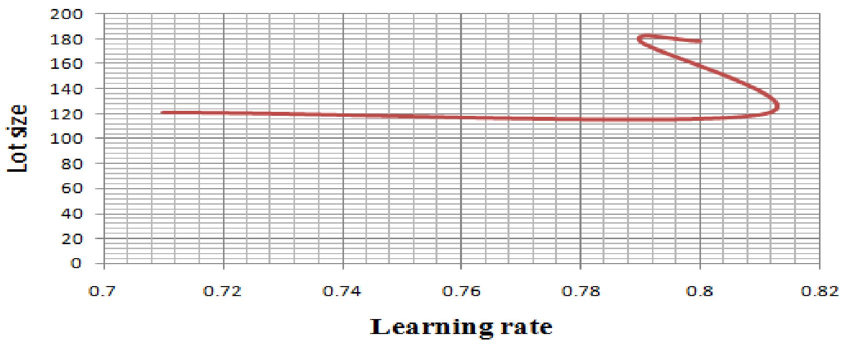

Figure 1: Effect of the learning on the lot size under fuzzy environment

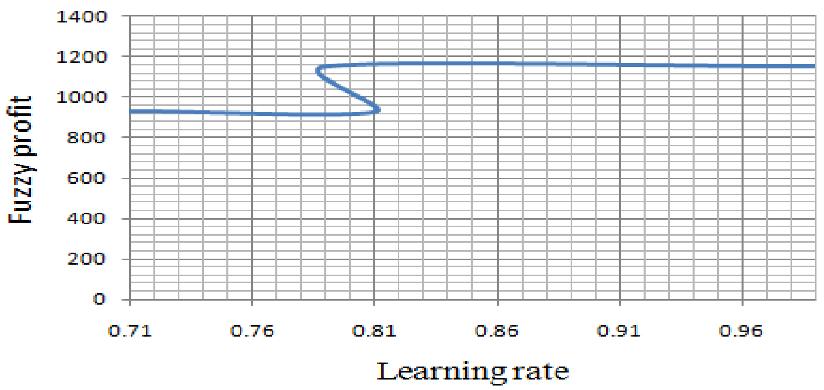

Figure 2: Effect of the learning on the fuzzy profit under fuzzy environment

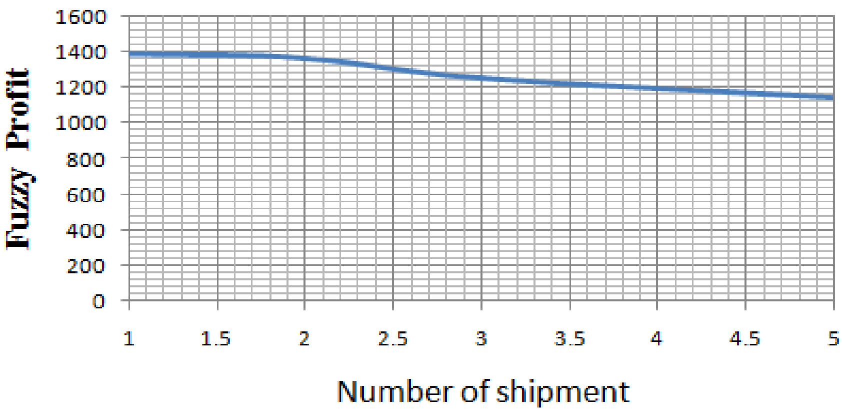

Figure 3: Effect of the shipment on the fuzzy profit under fuzzy environment

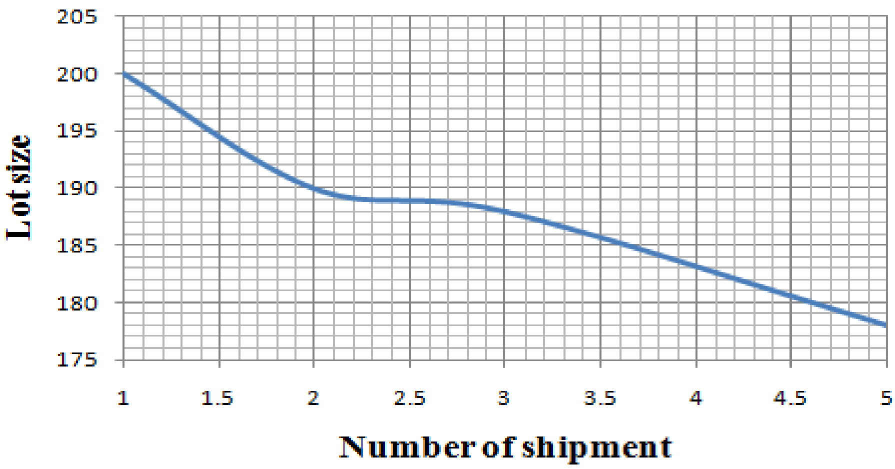

Figure 4: Effect of the shipment on the lot size under fuzzy environment

Observation and managerial insights

- ***Effect of the shipment***

We observed from Table 1 that as the number of shipments increases, cycle time and selling price increase, whereas lot size and profit decrease, whereas advertisement efforts remain constant from 1 to 5. Consequently, the buyer receives more information for the exercise of shipment, as shown in Figures 3 and 4.

- ***Effect of deterioration rate***

When the deterioration rate increases, the selling price increases, but the lot size, cycle time, and profit of the system decrease. The property of the deterioration of the product needs to be known to the seller and buyer during the course of the business.

- ***Effect of shape of the demand rate***

If the value of the shape of the demand rate increases, then cycle time, advertisement effort, and selling price decrease, but the lot size and profit increase. Hence the role of this input parameter gave positive effect in this model.

- ***Effect of* Stock-dependent parameter**

If the value of the Stock-dependent parameter increases then cycle time, advertisement effort,lot size and profit increase whereas selling price decreases. Hence the role of this input parameter also gave positive response in this model for both the players.

- ***Effect of learning rate***

Basically, the effect of learning affects the lot size and profit/ total cost directly. In this the value of learning increases then lot size and profit increase but other decision variables are constant and also presented the Figure 1 and 2.

- ***Effect of fuzzy input parameters***

From Table table-1, the value of the fuzzy input increases as the value of the cycle time, selling price, and advertisement effort increase, but the lot size and profit decrease. Hence, the fuzzy input gave a positive effect for the decision maker.

8. CONCLUSION

We developed a fuzzy based inventory model with advertisement efforts for deteriorating items under learning concept which is more applicable for the industrial sector, firms as well as online market. The effect of learning, advertisement efforts and fuzzy concept gave more applicable results for the business sector and also described the lot size when holding cost is imprecise in nature. Importance of the supporting input parameters also represented through numerical example as well as sensitivity analysis. This paper inquires an inventory model with constant deterioration and the mandate is presumed to be reliant on advertisement effect, product selling price, greening efforts, and stock level. And all-unit quantity discount is considered here, which increases the rate of consumption. After numerical illustrations, it is shown that total revenue rises when the advertisement factor is included in the demand function. A sensitivity table is also established to show the reliance of the resulting variables and the profit function on different inventory parameters. Some more results are also included in the observations and managerial insight. The present paper can be extended to include trade credit policy, shortages, and partial backlogging cases.

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