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Buyer-Seller Coordination Model for fuzzy deteriorating items

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Abstract

A coordinated buyer-seller inventory model for fuzzy deteriorating items is developed under uncertain cost and demand information. The model is developed using quantity discount policy under incomplete information environment. It is observed that the solution is very close to that when one has complete information. Fuzzy set theory is used to handle the uncertainty in the input parameters. The model is analyzed using triangular membership function. The model is illustrated with a suitable numerical example
Keywords: Buyer-Seller Co-ordination, Quantity discount, Fuzzy Deterioration, Fuzzy set theory..

1 Introduction and related literature

Coordination in supply chain management has gained much more attention than ever before. A supply chain refers to the production and distribution process from raw materials to finished goods. A supply chain includes of raw material suppliers through end users. Every party in a supply chain is usually an independent business. Those businesses have their own objectives, interest and perspectives of demand forecast. They try to gain competitive advantages, to maximize their profit. However, the individual objectives and interest may conflict with those of others. The conflicts limit the competitiveness of every company and worsen the performance of supply chain. Therefore, when companies face intense competition, companies can't fully use their

competitive advantage. To avoid such situation, many companies have realized the importance of coordination.

The single buyer and the single vendor together constitute a simple two-stage supply chain and form the basic building block of any supply chain. An effective supply chain network requires a co-operative relationship between the buyer and the vendor. Occurrence of conflicting objectives of different players leads to inefficiency in the supply chain. So the development of a suitable mechanism to co-ordinate different activities that are controlled by different players of the chain for attaining the goal is an important issue in supply chain management.

According to Battacharya et al. (1995), the supply chain members coordinate by sharing information regarding demand, orders, inventory etc. Information sharing is used, in effect, to integrate the entire value chain into one longer chain.

Munson and Rosenblatt (1998) have mentioned that 'quantity discount is the most effective mechanism of co-ordination in Business. The quantity discount policy was initiated with the focus to entice the buyers to procure at a larger quantity and thereby reducing the supplier's total operating cost. According to the Munson and Rosenblatt (1998) literature on supply chain co-ordination through quantity discounts under complete information can be broadly divided into the following three categories. They are (i) buyer's perspective models (ii) seller's perspective models and (iii) joint buyer-seller perspective models.

In the class of buyer's perspective models, Abad (1988); Chakravarty (1984, 1985, 1986); Bassok and Anupindi (1997) described their models taking the common assumptions that the quantity discount schedule is already available and in that situation buyers have to optimize their own requirement.

On the other hand, in the seller's perspective models, Banerjee (1986); Goyal (1987, 1987); Hwang and Kim (1986); Kim and Hwang (1988); Lee and Rosenblatt (1986); Monahan (1984); Parlar and Wang (1994); Rosenblatt and Lee (1985); Weng and wong (1993) have derived quantity discount pricing schemes that induce the buyers to change their order quantities from that of undiscounted price.

In the third class means joint buyer-seller models, Abad (1994, 2003); Banerjee (1986); Chakravarty and Martin (1988); Kohli and Perk (1989); Weng (1995); Munson and Rosenblatt (2001); Yang and Wee (2001); Viswanathan and Wang (2003); Qin,Tang and Guo (2007) derived models with quantity discount strategies to maximize system profit in such a way that no member ends up with a lesser profit as compared to the earlier non co-ordinated approach.

Li and Liu (2006); Shin and Benton (2006) and Zhou and Li (2007) have considered a probabilistic annual demand while the other authers have assumed mostly deterministic demand.

In all these models, one common assumption taken is that, the seller has complete information on the cost structure (ordering cost plus holding cost) of the buyer. But this assumption is unlikely in many real situations. Sometimes independent players of supply chain do not reveal their complete information. So in many cases information available in supply chain is incomplete. So now quantity-discount policy can be used to make a meaningful co-ordination between different supply chain players is an important concept.

To analyze the model with uncertainty, fuzzy set theory is highly effective. Zadeh (1965) developed the concept of fuzzy sets with uncertainty. Park (1987) developed an EOQ model with uncertain cost and used trapezoidal fuzzy numbers for inventory cost.

Yao and Lee (1996) considered EOQ model with shortages and fuzzy order quantity. Vujasevic M et al. (1996) developed EOQ model for single player with fuzzy inventory holding cost and fuzzy ordering cost. Chen et al. (1996) analyzed a fuzzy inventory model with backorder.

Hsieh (2002) has introduced two fuzzy inventory models with fuzzy parameters and derived the optimal production quantity by using graded mean integration representation method and extended Lagrangean method. Gen et al. (1997); Lee and Yao (1998, 1999); Lin and Yao (2000); Yao and Chang (2000); Yao and Su (2000) discussed different inventory problems that consider inventory with backorder, inventory without backorder and production inventory in the fuzzy sense.

Yao and Chiang (2003) considered an inventory model without backorder where total demand and holding cost were assumed to be fuzzy in nature and the authors had used different methods to derive total cost.

Mahata et al. (2005) have investigated the joint economic lot size (JELS) model for both purchaser and vendor in fuzzy sense and extended Banerjee's (1986) model with the assumption that the order quantity for the purchaser is a fuzzy variable while the rest of the parameters are deterministic.

Sarmaha et al. (2006) delivered that the coordination under limited information sharing is an important issue of concern to be studied for the decentralized supply chain.

From above it is found that there has been significant development of inventory models under uncertainty using the concept of fuzzy sets. Only Lam and Wong (1996); Corbett and Groote (2000) and Eric Sucky (2006) gives the issue of developing quantity discount policy for a buyer-vendor integrated system under uncertainty.

Lam and Wong (1996) have extended Dolan's (1978) deterministic model in a fuzzy sense. Eric Sucky (2006) has addressed the two-stage supply chain co-ordination problem under incomplete information with an assumption that the buyers could have only two sets of predefined cost structures and the seller does not know in which particular cluster does a particular buyer belongs.

Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage and loss of entity or loss of marginal value of a commodity that results in decreasing usefulness from the original one. IC chip, blood, fish, strawberries, alcohol, gasoline, radioactive chemicals and grain products are examples of deteriorating items.

Mandal, Roy, Maiti (1998) derive a fuzzy inventory model of deteriorating items with stock-dependent demand and limited storage space. Roy and Maiti (1998) derive multi-objective inventory models of deteriorating items with some constraints in a fuzzy environment. Yang and Wee (2000) used a heuristic approach to develop an integrated vendor-buyer inventory model for deteriorating items. Wee and Chung (2003) used a genetic algorithm to derive a VMI (vendor Managed Inventory) system for deteriorating items. Das and Roy and Maiti (2004) derive a buyer-seller fuzzy inventory model for a deteriorating item with discount.

Sinha and Sarmah (2008) derived Vendor-Buyer co-ordination using fuzzy parameters. They used triangular membership function to analyze the model. In this paper, an attempt is made to formulate joint buyer-seller inventory model for fuzzy deteriorating items under fuzzy environment. The units in inventory deteriorate at a constant rate. The shortage's at the buyer's end is not allowed. The uncertainty in input parameters is considered using fuzzy theory. The triangular membership function has been used to analyze the model. Finally the model is supported with a numerical example.

2 Assumptions and Notations

The Mathematical model is derived using following notations and assumptions.

2.1 Notations

A_b Ordering cost of the buyer in crisp (actual)

$\tilde{A}_b$ Ordering cost of the buyer in fuzzy (estimated)

A_v Set up cost of the seller

R Annual demand (actual)

$\tilde{R}$ Annual demand (estimated)

h_b Inventory holding cost per unit per year of the buyer in crisp (actual)

$\tilde{h}_b$ Inventory holding cost per unit per year of the buyer in fuzzy (estimated)

C_v Per unit production cost of the seller

C_b Un-discounted per unit sell price of the seller

P_d Discounted per unit sell price of the seller

Q Joint economic lot size (JELS) of the system

Q^* Economic order quantity (EOQ) of the buyer

T Cycle time

T^* Buyer's optimal cycle time

P_b Per unit sell-price of the buyer

θ Deterioration rate in crisp (actual)

$\tilde{\theta}$ Deterioration rate in fuzzy (estimated)

B_{nc}^* Buyer's profit when discount is not offered (non-coordinated policy)

V_{nc}^* Seller's profit when discount is not offered (non-coordinated policy)

π_B Profit function for the buyer when discount is not offered

B_c Buyer's profit when discount is offered (coordinated policy)

V_c Seller's profit when discount is offered (coordinated policy)

b Benefit of the buyer

v Benefit of the seller

J Total coordinated benefit for both the seller and the buyer

2.2 Assumptions

- The single-seller single-buyer supply chain is considered.
- The inventory system under consideration deals with single items.
- The seller follows lot-for-lot policy.
- The shortages at the buyer's end are not allowed.
- The deteriorate units can neither be replaced nor repaired during the cycle time.
- The incomplete information is available to the seller.
- The seller is dominant player.

3 Mathematical Model

3.1 Non-Coordinated Policy

For buyer,

Sales Revenue, $SR = P_b Q$

Purchase cost, $PC = C_b Q$

Inventory holding cost, $IHC = \tilde{h}_b \int_0^T I(t)dt$

$$= \tilde{h}_b \int_0^T \frac{\tilde{R}}{\tilde{\theta}} \left\{ e^{\tilde{\theta}(T-t)} - 1 \right\} dt$$

$$= \tilde{h}_b \frac{\tilde{R}}{\tilde{\theta}^2} \left\{ e^{\tilde{\theta}T} - \tilde{\theta}T - 1 \right\}$$

$$= \tilde{h}_b \tilde{R} \left\{ \frac{T^2}{2} + \frac{\tilde{\theta}T^3}{6} \right\}$$

Ordering cost, $OC = \frac{\tilde{A}_b}{T}$

The profit function of the buyer is,

$$\pi_B = (SR - PC - IHC - OC)\frac{1}{T}$$

$$= \frac{1}{T}\left(P_b - C_b\right)\left[\tilde{R}T + \frac{\tilde{R}\tilde{\theta}T^2}{2}\right] - \frac{\tilde{h}_b\tilde{R}}{T}\left\{\frac{T^2}{2} + \frac{\tilde{\theta}T^3}{6}\right\} - \frac{\tilde{A}_b}{T}$$

$$\pi_B = (P_b - C_b)\tilde{R} + \frac{\tilde{R}\tilde{\theta}T}{2}(P_b - C_b) - \frac{\tilde{h}_b\tilde{R}T}{2} - \frac{\tilde{A}_b}{T} - \frac{\tilde{h}_b\tilde{R}\tilde{\theta}T^2}{6} \quad ...(1)$$

with incomplete information about the annual demand, inventory holding cost and ordering cost of the buyer, the buyer's optimal cycle time which maximizes (1) is (ignoring term with θT^2)

Now from (1),

$$\frac{d\pi_B}{dT} = \frac{\tilde{R}\tilde{\theta}}{2}(P_b - C_b) - \frac{\tilde{h}_b\tilde{R}}{2} - \frac{\tilde{A}_b}{T^2} = 0$$

$$\frac{\tilde{A}_b}{T^2} = \frac{\tilde{h}_b\tilde{R}}{2} - \frac{\tilde{R}\tilde{\theta}}{2}(P_b - C_b)$$

$$T^2 = \frac{2\tilde{A}_b}{\tilde{R}\left(\tilde{h}_b + \tilde{\theta}\left(C_b - P_b\right)\right)}$$

So the buyer's optimal cycle time is given by,

$$T^* = \sqrt{\frac{2\tilde{A}_b}{\tilde{R}\left(\tilde{h}_b + \tilde{\theta}\left(C_b - P_b\right)\right)}} \quad ...(2)$$

Thus

$$\left(B\right)_{nc} = B^*_{nc} = (P_b - C_b)\left[\tilde{R} + \frac{\tilde{R}\tilde{\theta}T^*}{2}\right] - \frac{\tilde{h}_b\tilde{R}T^*}{2} - \frac{\tilde{A}_b}{T} - \frac{\tilde{h}_b\tilde{R}\tilde{\theta}T^{*2}}{6} \quad ...(3)$$

When no discount is offered and the buyer's cycle time (T^*) is known, the seller's profit is given by

$$(V)_{nc}^{*} = (C_b - C_v)\tilde{R} - \frac{A_v}{T^*} \quad ...(4)$$

3.2 Coordinated Policy

When seller offers discounted unit-price P_d, then for a cycle time *T*, the buyer's profit is given by

$$B_c = (P_b - P_d)\tilde{R} + \frac{(P_b - C_b)\tilde{R}\tilde{\theta}T}{2} - \frac{\tilde{h}_b\tilde{R}T}{2} - \frac{\tilde{A}_b}{T} - \frac{\tilde{h}_b\tilde{R}\tilde{\theta}T^2}{6} \quad ...(5)$$

and the profit equation of the seller is

$$V_c = (P_d - C_v)\tilde{R} - \frac{A_v}{T} \quad ...(6)$$

Then, the benefit of the buyer under available discount is,

$$b = B_c - B_{nc}^{*}$$

$$= (C_b - P_d)\tilde{R} + \frac{(P_b - C_b)\tilde{R}\tilde{\theta}(T - T^*)}{2} - \frac{\tilde{h}_b\tilde{R}(T - T^*)}{2} - \frac{\tilde{h}_b\tilde{R}\tilde{\theta}(T^2 - T^{*2})}{6} + \tilde{A}_b\left(\frac{1}{T^*} - \frac{1}{T}\right) \quad ...(7)$$

and the benefit of the seller is,

$$v = (V_c - V_{nc}^{*})$$

$$= A_v\left(\frac{1}{T^*} - \frac{1}{T}\right) - (C_b - P_d)\tilde{R} \quad ...(8)$$

The objective is to maximize total coordinated benefit for both the seller and the buyer.

$$J = b + v$$

$$= (\tilde{A}_b + A_v)\left(\frac{1}{T^*} - \frac{1}{T}\right) + \frac{(P_b - C_b)\tilde{R}\tilde{\theta}(T - T^*)}{2} - \frac{\tilde{h}_b\tilde{R}(T - T^*)}{2} - \frac{\tilde{h}_b\tilde{R}\tilde{\theta}(T^2 - T^{*2})}{6} \quad ...(9)$$

Under the assumption that the seller offers his buyer a certain level of coordination benefit, we have

$$b = \gamma v \quad ...(10)$$

where $\gamma \geq 0$ is pre-defined constant to share the benefit between two players. From (7), (8) and (10),

$$P_d = C_b + \frac{(P_b - C_b)\tilde{\theta}(T - T^*)}{2(1+\gamma)} - \frac{\tilde{h}_b(T - T^*)}{2(1+\gamma)} - \frac{\tilde{h}_b\tilde{\theta}(T^2 - T^{*2})}{6(1+\gamma)} - \frac{(\tilde{A}_b - \gamma A_v)}{\tilde{R}(1+\gamma)}\left(\frac{1}{T} - \frac{1}{T^*}\right) \quad (11)$$

The optimum value of T can be obtained by setting $\frac{\partial J}{\partial T} = 0$ which gives

$$T = \sqrt{\frac{2(\tilde{A}_b + A_v)}{\tilde{R}(\tilde{h}_b + \tilde{\theta}(C_b - P_b))}} \quad ...(12)$$

With obtained T, one can compute total coordination benefit J and the seller's discounted selling price (P_d) for given value of ν. In next section procedure is outlined.

4 Solution Procedure

From (12), the optimal T is as follows,

$$T = \sqrt{\frac{2(\tilde{A}_b + A_v)}{\tilde{R}(\tilde{h}_b + \tilde{\theta}(C_b - P_b))}}$$

However, $\tilde{A}_b, \tilde{h}_b$, $\tilde{R}$ and $\tilde{\theta}$ are not known precisely and let $\tilde{A}_b, \tilde{h}_b, \tilde{R}$ and $\tilde{\theta}$ be defined by triangular fuzzy numbers such that

$$\tilde{A}_b = [a_1, a_2, a_3],\ \tilde{h}_b = [b_1, b_2, b_3], \tilde{R} = [r_1, r_2, r_3] \text{ and } \tilde{\theta} = [\theta_1, \theta_2, \theta_3]$$

where, $(a_1 < a_2 < a_3)$, $(b_1 < b_2 < b_3)$, $(r_1 < r_2 < r_3)$ and $(\theta_1 < \theta_2 < \theta_3)$. We apply arithmetic operators on fuzzy quantities and then de-fuzzify the same to convert them to crisp output.

The membership functions of $\mu_{\tilde{A}_b}(\tilde{A}_b), \mu_{\tilde{h}_b}(\tilde{h}_b), \mu_{\tilde{R}}(\tilde{R})$ and $\mu_{\tilde{\theta}}(\tilde{\theta})$ are defined as follows

$$\mu_{\tilde{A}_b}(\tilde{A}_b)=\begin{cases}0 if\ A_b < a_1 \\ \dfrac{A_b - a_1}{a_2 - a_1} if\ a_1 \le A_b < a_2 \\ \dfrac{a_3 - A_b}{a_3 - a_2} if\ a_2 \le A_b < a_3 \\ 0 if\ A_b \ge a_3\end{cases} \quad ...(A)$$

$$\mu_{\tilde{h}_b}(\tilde{h}_b)=\begin{cases}0 if\ h_b < b_1 \\ \dfrac{h_b - b_1}{b_2 - b_1} if\ b_1 \le h_b < b_2 \\ \dfrac{b_3 - h_b}{b_3 - b_2} if\ b_2 \le h_b < b_3 \\ 0 if\ h_b \ge b_3\end{cases} \quad ...(B)$$

$$\mu_{\tilde{R}}(\tilde{R})=\begin{cases}0 if R < r_1 \\ \dfrac{R - r_1}{r_2 - r_1} if\ r_1 \le R < r_2 \\ \dfrac{r_3 - R}{r_3 - r_2} if\ r_2 \le R < r_3 \\ 0 if R \ge r_3\end{cases} \quad ...(C)$$

$$\mu_{\tilde{\theta}}(\tilde{\theta})=\begin{cases}0\ if\theta < \theta_1 \\ \dfrac{\theta - \theta_1}{\theta_2 - \theta_1} if\ \theta_1 \le \theta \le \theta_2 \\ \dfrac{\theta_3 - \theta}{\theta_3 - \theta_2} if\ \theta_2 \le \theta \le \theta_3 \\ 0 if\theta \ge \theta_3\end{cases} \quad ...(D)$$

Now using the concept of '$\alpha - cut$'method, from (A), (B), (C) and (D),

$${}^{\alpha}A_b = \left[\alpha(a_2 - a_1) + a_1, a_3 - \alpha(a_3 - a_2)\right] \quad \text{for}\ \alpha \in [0,1],$$

$${}^{\alpha}h_b = \left[\alpha(b_2 - b_1) + b_1, b_3 - \alpha(b_3 - b_2)\right] \quad \text{for}\ \alpha \in [0,1],$$

$${}^{\alpha}R = \left[\alpha(r_2 - r_1) + r_1, r_3 - \alpha(r_3 - r_2)\right] \qquad \text{for } \alpha \in [0,1],$$

$${}^{\alpha}\theta = \left[\alpha(\theta_2 - \theta_1) + \theta_1, \theta_3 - \alpha(\theta_3 - \theta_2)\right] \qquad \text{for } \alpha \in [0,1].$$

4.1 Derivation of T^*

It is known that, $T^* = \sqrt{\dfrac{2\tilde{A}_b}{\tilde{R}\left(\tilde{h}_b + \tilde{\theta}\left(C_b - P_b\right)\right)}}$. From inequality

$$\left\{\frac{a_3 - \alpha(a_3 - a_2)}{\alpha(b_2 - b_1) + b_1}\right\} \geq \left\{\frac{\alpha(a_2 - a_1) + a_1}{b_3 - \alpha(b_3 - b_2)}\right\} \qquad \text{for } \alpha \in [0,1]$$

Thus,

$${}^{\alpha}\left\{\frac{\tilde{A}_b}{\tilde{h}_b + (C_b - P_b)\tilde{\theta}}\right\} = \left[\begin{array}{l}\left\{\dfrac{\alpha(a_2 - a_1) + a_1}{b_3 - \alpha(b_3 - b_2) + (C_b - P_b)\{\theta_3 - \alpha(\theta_3 - \theta_2)\}}\right\}, \\ \left\{\dfrac{a_3 - \alpha(a_3 - a_2)}{\alpha(b_2 - b_1) + b_1 + (C_b - P_b)\{\alpha(\theta_2 - \theta_1) + \theta_1\}}\right\}\end{array}\right]$$

The value of ${}^{\alpha}\left[T^{*2}\right] = {}^{\alpha}\left[\dfrac{2\tilde{A}_b}{\tilde{R}\left(\tilde{h}_b + \left(C_b - P_b\right)\tilde{\theta}\right)}\right]$ is derived as follows,

$${}^{\alpha}\left[T^{*2}\right] = \left[\begin{array}{l}\dfrac{2}{\left\{r_3 - \alpha(r_3 - r_2)\right\}}\dfrac{\alpha(a_2 - a_1) + a_1}{\left\{b_3 - \alpha(b_3 - b_2)\right\} + \left(C_b - P_b\right)\{\theta_3 - \alpha(\theta_3 - \theta_2)\}}, \\ \dfrac{2}{\left\{\alpha(r_2 - r_1) + r_1\right\}}\dfrac{a_3 - \alpha(a_3 - a_2)}{\{\alpha(b_2 - b_1) + b_1\} + \left(C_b - P_b\right)\{\alpha(\theta_2 - \theta_1) + \theta_1\}}\end{array}\right] \qquad \text{...(13)}$$

We put $\alpha = 0$ and $\alpha = 1$ in (13) and obtain an approximate triangular fuzzy number for T^* as below

$$T^* = \left[\sqrt{\frac{2a_1}{r_3(b_3 + (C_b - P_b)\theta_3)}}, \sqrt{\frac{2a_2}{r_2(b_2 + (C_b - P_b)\theta_2)}}, \sqrt{\frac{2a_3}{r_1(b_1 + (C_b - P_b)\theta_1)}}\right]$$

$$= \left[t_1^*, t_2^*, t_3^*\right] \qquad \text{....(14)}$$

where $t_1^* = \sqrt{\frac{2a_1}{r_3(b_3+(C_b-P_b)\theta_3)}}$, $t_2^* = \sqrt{\frac{2a_2}{r_2(b_2+(C_b-P_b)\theta_2)}}$,

$$t_3^* = \sqrt{\frac{2a_3}{r_1(b_1+(C_b-P_b)\theta_1)}}$$

Thus, the membership function for $\mu_{\tilde{T}^*}(\tilde{T}^*)$ is given as,

$$\mu_{\tilde{T}^*}(\tilde{T}^*) = \begin{cases} 0 if T^* < t_1^* \\ \frac{T^* - t_1^*}{t_2^* - t_1^*} if t_1^* \le T^* < t_2^* \\ \frac{t_3^* - T^*}{t_3^* - t_2^*} if t_2^* \le T^* < t_3^* \\ 0 if T^* \ge t_3^* \end{cases} \quad ...(D)$$

T^* is de-fuzzified to a crisp value by the 'centre of gravity' method. The de-fuzzified T^* is found as,

$$T^* = defuzz(\tilde{T}^*) = \frac{\int_R T^* \mu_{\tilde{T}^*}(\tilde{T}^*) dT^*}{\int_R \mu_{\tilde{T}^*}(\tilde{T}^*) dT^*} \quad ...(15)$$

4.2 Derivation of optimal *T*

It is known that, $T = \sqrt{\frac{2(\tilde{A}_b + A_v)}{\tilde{R}(\tilde{h}_b + \tilde{\theta}(C_b - P_b))}}$

Since, A_v is a crisp element, it may be represented as,

$${}^{\alpha}A_v = [A_v, A_v] \text{ For } \alpha \in [0,1],$$

Using arithmetic operators,

$$^{\alpha}(\tilde{A}_b + A_v) = \left[\{\alpha(a_2 - a_1) + a_1 + A_v\}, \{a_3 + A_v - \alpha(a_3 - a_2)\}\right]$$

for $\alpha \in [0,1]$ ...(16)

From inequality,

$$\left\{\frac{\{(a_3 + A_v) - \alpha(a_3 - a_2)\}}{\alpha(b_2 - b_1) + b_1}\right\} \geq \left\{\frac{\alpha(a_2 - a_1) + a_1 + A_v}{b_3 - \alpha(b_3 - b_2)}\right\} \text{ for } \alpha \in [0,1] \quad ...(17)$$

Thus, $$^{\alpha}\left(\frac{\tilde{A}_b + A_v}{\tilde{h}_b + \tilde{\theta}(C_b - P_b)}\right) = \left[\begin{matrix}\left\{\frac{\alpha(a_2 - a_1) + a_1 + A_v}{b_3 - \alpha(b_3 - b_2) + \{\theta_3 - \alpha(\theta_3 - \theta_2)\}(C_b - P_b)}\right\}, \\ \left\{\frac{(a_3 + A_v) - \alpha(a_3 - a_2)}{\alpha(b_2 - b_1) + b_1 + \{\alpha(\theta_2 - \theta_1) + \theta_1\}(C_b - P_b)}\right\}\end{matrix}\right] \quad ...(18)$$

and, the value of $$^{\alpha}\left[T^2\right] = {}^{\alpha}\left[\frac{2}{\tilde{R}}\left\{\frac{\tilde{A}_b + A_v}{\tilde{h}_b + \tilde{\theta}(C_b - P_b)}\right\}\right]$$ is derived as follows,

$$= \left[\begin{matrix}\frac{2}{\{r_3 - \alpha(r_3 - r_2)\}}\left\{\frac{\alpha(a_2 - a_1) + a_1 + A_v}{b_3 - \alpha(b_3 - b_2) + \{\theta_3 - \alpha(\theta_3 - \theta_2)\}(C_b - P_b)}\right\}, \\ \frac{2}{\{\alpha(r_2 - r_1) + r_1\}}\left\{\frac{(a_3 + A_v) - \alpha(a_3 - a_2)}{\alpha(b_2 - b_1) + b_1 + \{\alpha(\theta_2 - \theta_1) + \theta_1\}(C_b - P_b)}\right\}\end{matrix}\right] \quad ...(19)$$

We put $\alpha = 0$ and $\alpha = 1$ in (19) and obtain an approximate triangular fuzzy number for T as below

$$T = \left[\sqrt{\frac{2(a_1 + A_v)}{r_3\{b_3 + \theta_3(C_b - P_b)\}}}, \sqrt{\frac{2(a_2 + A_v)}{r_2\{b_2 + \theta_2(C_b - P_b)\}}}, \sqrt{\frac{2(a_3 + A_v)}{r_1\{b_1 + \theta_1(C_b - P_b)\}}}\right]$$

$$= [t_1, t_2, t_3] \quad ...(20)$$

where $t_1 = \sqrt{\frac{2(a_1 + A_v)}{r_3\{b_3 + \theta_3(C_b - P_b)\}}}$, $t_2 = \sqrt{\frac{2(a_2 + A_v)}{r_2\{b_2 + \theta_2(C_b - P_b)\}}}$,

$$t_3 = \sqrt{\frac{2(a_3 + A_v)}{r_1\{b_1 + \theta_1(C_b - P_b)\}}}$$

The membership function for $\mu_{\tilde{T}}(T)$ is derived as,

$$\mu_{\tilde{T}}(T)=\begin{cases}0, if T<t_1\\ \dfrac{T-t_1}{t_2-t_1} if\, t_1\le T<t_2\\ \dfrac{t_3-T}{t_3-t_2} if\, t_2\le T<t_3\\ 0, if T\ge t_3\end{cases} \quad ...(E)$$

Similarly, T is de-fuzzified to a crisp value by the 'centre of gravity' method. The de-fuzzified T is found as,

$$T=defuzz(\tilde{T})=\frac{\int_R T\mu_{\tilde{T}}(T)dT}{\int_R \mu_{\tilde{T}}(T)dT} \quad ...(21)$$

4.3 Derivation of discounted sell-price P_d

From (11),

$$P_d=C_b-\left[\left(\gamma A_v-\tilde{A}_b\right)\frac{g_1}{\tilde{R}}-\left\{\tilde{h}_b+\left(C_b-P_b\right)\tilde{\theta}\right\}g_2-\frac{\tilde{h}_b\tilde{\theta}}{3}(T^*+T)g_2\right]$$

Where $g_1=\left(\frac{1}{T^*}-\frac{1}{T}\right)\frac{1}{(1+\gamma)}$, $g_2=\frac{(T^*-T)}{2(1+\gamma)}$

$$= C_b-\left[\left(\gamma A_v-\tilde{A}_b\right)\frac{g_1}{\tilde{R}}-\left\{\tilde{h}_b+\tilde{\theta}(C_b-P_b)+\frac{\tilde{h}_b\tilde{\theta}}{3}(T^*+T)\right\}g_2\right]$$

$$= C_b-\left[(\gamma A_v-\tilde{A}_b)\frac{g_1}{\tilde{R}}-\left\{\tilde{h}_b+\tilde{\theta}\left[\tilde{h}_b g_3+(C_b-P_b)\right]\right\}g_2\right]$$

where, $g_3=\frac{1}{3}(T^*+T)$

$$\overset{\alpha}{\frac{(\gamma A_v - \tilde{A}_b)g_1}{\tilde{R}}} = \left[\left\{\frac{(\gamma A_v - b_1) - \alpha(b_2 - b_1)}{r_3 - \alpha(r_3 - r_2)}\right\}g_1, \left\{\frac{(\gamma A_v - b_3 + \alpha(b_3 - b_2)}{\alpha(r_2 - r_1) + r_1}\right\}g_1\right]$$

$$= \begin{bmatrix} \{\alpha(b_2 - b_1) + b_1\}\left[1 + \{\alpha(\theta_2 - \theta_1) + \theta_1\}g_3\right] + (C_b - P_b)\{\alpha(\theta_2 - \theta_1) + \theta_1\}, \\ \{b_3 - \alpha(b_3 - b_2)\}\left[1 + \{\theta_3 - \alpha(\theta_3 - \theta_2)\}g_3\right] + (C_b - P_b)\{\theta_3 - \alpha(\theta_3 - \theta_2)\} \end{bmatrix}$$

$$\overset{\alpha}{(P_d)} = \begin{bmatrix} C_b - \left\{\frac{(\gamma A_v - b_3) + \alpha(b_3 - b_2)}{\alpha(r_2 - r_1) + r_1}\right\}g_1 + \{\alpha(b_2 - b_1) + b_1\}\left[1 + \{\alpha(\theta_2 - \theta_1) + \theta_1\}g_3\right]g_2 \\ +(C_b - P_b)\{\alpha(\theta_2 - \theta_1) + \theta_1\}g_2, \\ C_b - \left\{\frac{(\gamma A_v - b_1) - \alpha(b_2 - b_1)}{r_3 - \alpha(r_3 - r_2)}\right\}g_1 + \{b_3 - \alpha(b_3 - b_2)\}\left[1 + \{\theta_3 - \alpha(\theta_3 - \theta_2)\}g_3\right]g_2 \\ +(C_b - P_b)\{\theta_3 - \alpha(\theta_3 - \theta_2)\}g_3 \end{bmatrix} \quad \text{...(22)}$$

We put $\alpha = 0$ and in (22) and obtain an approximate triangular fuzzy number for P_d. Thus,

$$P_d = \begin{bmatrix} C_b - \frac{(\gamma A_v - b_3)}{r_1}g_1 + b_1(1 + \theta_1 g_3)g_2 + (C_b - P_b)\theta_1 g_2, \\ C_b - \frac{(\gamma A_v - b_2)}{r_2}g_1 + b_2(1 + \theta_2 g_3)g_2 + (C_b - P_b)\theta_2 g_2, \\ C_b - \frac{(\gamma A_v - b_1)}{r_3}g_1 + b_3(1 + \theta_3 g_3)g_2 + (C_b - P_b)\theta_3 g_2 \end{bmatrix}$$

$$= \left[P_{d_1}, P_{d_2}, P_{d_3}\right] \quad \text{...(23)}$$

where $P_{d_1} = \left\{C_b - \frac{(\gamma A_v - b_3)}{r_1}g_1 + b_1(1 + \theta_1 g_3)g_2 + (C_b - P_b)\theta_1 g_2,\right\}$,

$$P_{d_2} = \left\{C_b - \frac{(\gamma A_v - b_2)}{r_2}g_1 + b_2(1 + \theta_2 g_3)g_2 + (C_b - P_b)\theta_2 g_2,\right\},$$

$$P_{d_3} = \left\{C_b - \frac{(\gamma A_v - b_1)}{r_3}g_1 + b_3(1 + \theta_3 g_3)g_2 + (C_b - P_b)\theta_3 g_2\right\}.$$

From (23), the membership function for $\mu_{\tilde{P}_d}(P_d)$ is derived as,

$$\mu_{\tilde{P}_d}(P_d)=\begin{cases}0, if\, P_d < P_{d_1}\\ \dfrac{P_d - P_{d_1}}{P_{d_2} - P_{d_1}}, if\, P_{d_1} \le P_d < P_{d_2}\\ \dfrac{P_{d_3} - P_d}{P_{d_3} - P_{d_2}}, if\, P_{d_2} \le P_d < P_{d_3}\\ 0, if\, P_d \ge P_{d_3}\end{cases} \quad ...(F)$$

P_d is de-fuzzified to a crisp value by the method of 'centre of gravity' method. The de-fuzzified P_d is found as,

$$P_d = defuzz(\tilde{P}_d)=\frac{\int_R P_d \mu_{\tilde{P}_d}(P_d)dP_d}{\int_R \mu_{\tilde{P}_d}(P_d)dP_d} \quad ...(24)$$

4.4 Derivation of total coordination benefit (*J*)

It has already been derived that,

$$J = (\tilde{A}_b + A_v)\left(\frac{1}{T^*}-\frac{1}{T}\right)+\frac{(P_b - C_b)\tilde{R}\theta(T-T^*)}{2}-\frac{\tilde{h}_b\tilde{R}(T-T^*)}{2}-\frac{\tilde{h}_b\tilde{R}\theta(T^2-T^{*2})}{6}$$

$$= (\tilde{A}_b + A_v)g_4-(P_b - C_b)\tilde{R}\tilde{\theta}g_5+\tilde{h}_b\tilde{R}g_5+\tilde{h}_b\tilde{R}\tilde{\theta}g_5g_6$$

where, $g_4=\left(\frac{1}{T^*}-\frac{1}{T}\right), g_5=\frac{(T^*-T)}{2}, g_6=\frac{(T^*+T)}{3}$

$$= (\tilde{A}_b + A_v)g_4+\tilde{R}\left\{\tilde{h}_b\left(1+g_6\tilde{\theta}\right)-\tilde{\theta}(P_b - C_b)\right\}g_5$$

Applying $\alpha - cut$ and arithmetic operators,

$$^{\alpha}\left[(\tilde{A}_b + A_v)g_4\right]=\left[\left\{(A_v + a_1)+\alpha(a_2 - a_1)\right\}g_4, \left\{(A_v + a_3)-\alpha(a_3 - a_2)\right\}g_4\right]$$

For $\alpha \in [0,1]$,

$$^{\alpha}\left[\tilde{R}\left\{\tilde{h}_b\left(1+g_6\tilde{\theta}\right)-\tilde{\theta}(P_b - C_b)\right\}\right]=$$
$$\left[\left\{\alpha(r_2 - r_1)+r_1\right\}\left[\left\{\alpha(b_2 - b_1)+b_1\right\}\left\{1+g_6\left[\alpha(\theta_2 - \theta_1)+\theta_1\right]\right\}-(P_b - C_b)\left\{\alpha(\theta_2 - \theta_1)+\theta_1\right\}\right]\right.$$
$$\left.\left\{r_3 - \alpha(r_3 - r_2)\right\}\left[\left\{b_3 - \alpha(b_3 - b_2)\right\}\left\{1+g_6\left[\theta_3 - \alpha(\theta_3 - \theta_2)\right]\right\}-(P_b - C_b)\left\{\theta_3 - \alpha(\theta_3 - \theta_2)\right\}\right]\right]$$

for $\alpha \in [0,1]$. Thus,

$$^{\alpha}(J) = \begin{bmatrix} \{(A_v + a_1) + \alpha(a_2 - a_1)\} g_4 + \left\{ \{\alpha(r_2 - r_1) + r_1\} \begin{bmatrix} \{\alpha(b_2 - b_1) + b_1\}\{1 + g_6[\alpha(\theta_2 - \theta_1) + \theta_1]\} \\ -(P_b - C_b)\{\alpha(\theta_2 - \theta_1) + \theta_1\} \end{bmatrix} \right\} g_5, \\ \{(A_v + a_3) - \alpha(a_3 - a_2)\} g_4 + \left\{ \{r_3 - \alpha(r_3 - r_2)\} \begin{bmatrix} \{b_3 - \alpha(b_3 - b_2)\}\{1 + g_6[\theta_3 - \alpha(\theta_3 - \theta_2)]\} \\ -(P_b - C_b)\{\theta_3 - \alpha(\theta_3 - \theta_2)\} \end{bmatrix} \right\} g_5 \end{bmatrix} \quad ...(25)$$

We put $\alpha = 0$ and $\alpha = 1$ in (25) and obtain an approximate triangular fuzzy number for J.

$$^{\alpha}(J) = \begin{bmatrix} (A_v + a_1) g_4 + r_1 \{(1 + g_6\theta_1) b_1 - (P_b - C_b)\theta_1\} g_5, \\ (A_v + a_2) g_4 + r_2 \{(1 + g_6\theta_2) b_2 - (P_b - C_b)\theta_2\} g_5, \\ (A_v + a_3) g_4 + r_3 \{(1 + g_6\theta_3) b_3 - (P_b - C_b)\theta_3\} g_5 \end{bmatrix}$$

$$= [j_1, j_2, j_3] \quad ...(26)$$

where, $j_1 = (A_v + a_1) g_4 + r_1 \{(1 + g_6\theta_1) b_1 - (P_b - C_b)\theta_1\} g_5,$

$$j_2 = (A_v + a_2) g_4 + r_2 \{(1 + g_6\theta_2) b_2 - (P_b - C_b)\theta_2\} g_5, \; j_2 =$$

$$j_3 = (A_v + a_3) g_4 + r_3 \{(1 + g_6\theta_3) b_3 - (P_b - C_b)\theta_3\} g_5 .$$

From (26), the membership function for $\mu_{\tilde{J}}(J)$ is derived as,

$$\mu_{\tilde{J}}(J) = \begin{cases} 0, if J < j_1 \\ \dfrac{J - j_1}{j_2 - j_1}, if\, j_1 \le J < j_2 \\ \dfrac{j_3 - J}{j_3 - j_2}, if\, j_2 \le J < j_3 \\ 0, if J \ge j_3 \end{cases} \quad ...(G)$$

J is de-fuzzified to a crisp value by the 'centre of gravity' method. The de-fuzzified J is found as,

$$J = defuzz(\tilde{J}) = \frac{\int_R J\mu_{\tilde{J}}(J)dJ}{\int_R \mu_{\tilde{J}}(J)dJ} \quad ...(27)$$

5 Numerical Example

In this part, firstly we assume that all the parameter values are known to the seller and according to this information we calculate the solution. After that we take A_b, h_b, R and θ to be asymmetric and define them by triangular fuzzy numbers and calculate the optimal parameters. After getting the solution, compare this solution with that of the solution derived under complete information scenario.

From Chakravarty and Martin (1988), we take the following data set (some additional values also) which is used for the numerical illustration.

Table 5.1 Data set under complete information

A_v	C_v	C_b	A_b	h_b	P_b	R	γ	θ
500	20	25	100	5	30	1000	1.0	0.30

5.1 Complete Information

Here, we derive the optimal solution under complete information scenario. The results are shown in the following table.

Table 5.2 Solution under complete information

Non-Coordinated Approach				*Coordinated Approach*					
T^*	B^*_{nc}	V^*_{nc}	Total	T	P_d	B_c	V_c	Total	J
0.239046	4149.05	2908.35	7057.40	0.585540	24.1660	4552.80	3312.10	7864.90	807.49

5.2 Incomplete Information

Here, we assume that the annual demand R, the ordering cost A_b, the holding cost h_b of the buyer and deterioration rate θ, are not known completely to the seller. The seller estimates the values by the following triangular fuzzy numbers based on his subjective judgment.

Table 5.3 Data set under incomplete information

A_b	h_b	R	θ
[80 100 120]	[3 5 7]	[800 1000 1200]	[0.10 0.30 0.50]

In this case, the optimal values are shown in the following table.

Table 5.4 Solution under incomplete information

T^*	T	P_d	J
0.258582	0.623919	24.0742	565.4917

If this optimal $\{T, P_d\}$ is implemented, the following table 5.5 shows the individual actual pay-offs.

Table 5.5 Actual pay-offs under incomplete information

B_c	V_c	Total	b	v	J
4576.33	3272.83	7849.16	427.279	364.476	791.7556

From table 5.2 and 5.5, it is seen that the individual pay-offs received by the buyer and the seller and the profit (J) is very close to that of the solution under complete information.

5.3 Sensitivity Analysis

Here, we have conducted a simulation test by generating random data for the imprecise variables with in a certain range as given below. The test is conducted with 2500 randomly generated cases and the optimal T and P_d are derived in each case which is again put in the original equation to find out the actual pay-offs for both buyer and the seller. Data set for the simulation test is given in the following Table 5.6 and table 5.7 shows the performance of the simulation test.

Table 5.6 Data set for simulation test

A_b	h_b	R	θ
[80 120]	[3 7]	[800 1200]	[0.10 0.50]

Table 5.7 Actual pay-offs under incomplete information: A Simulation test

Parameter	B	V	b	v	J	γ
Average	4147.83	2918.48	4549.07	6214.51	790.147	0.4983
Max.	5550.90	4937.16	5638.94	7923.88	1445.93	0.9994
Min.	2993.47	1343.54	3537.45	4537.23	-134.794	0.0001
Std. dev.	548.735	627.33	556.522	691.337	237.933	0.2868

6 Conclusion

In this paper, we used a quantity discount policy to coordinate a buyer and a seller under incomplete information for fuzzy deteriorating items. To model the problem we use fuzzy triangular membership function. Consider the annual demand, the ordering cost and the holding cost of the buyer and deterioration rate to be fuzzy, and we use the fuzzy arithmetic operators to compute the joint economic lot size. It is very well known that, the coordination benefit is maximum under complete information so it is important to share information for a meaningful coordination. In this paper we generate 2500 random cases and conduct a simulation test and we get a near optimal solution with incomplete information which is very close to that of the complete solution.

References

1. Abad P. (1988). "Determining optimal selling price and lot size when the supplier offers all-unit quantity discount", *Decision Sciences*, 19(3): 622-634.
2. Abad P. (1994). "Supplier pricing and lot sizing when demand is price sensitive", *European Journal of Operational Research*, 78(3): 334-354.
3. Abad P. (2003). "Optimal price and lot size when the supplier offers a temporary price reduction over an interval", *Computers and Operations Research*, 30(1): 63-74.
4. Bannerjee A. (1986). "A joint economic lot size model for purchaser and vendor", *Decision Sciences*, 17(3): 292-311.
5. Bassok Y. and Anupindi R. (1997). "Analysis of supply contracts with total minimum commitment", *IIE Transactions*, 29(5): 373-381.
6. Bhattacharya A. K., Coleman J. L., Brace G. (1995). "Re-positioning the supplier: an SME location inventory system", Working paper, Graduate School of Business, Columbia University, New York, NY.
7. Chakravarty A. (1984). "Joint inventory replenishments with group discounts based on invoice value", *Management Science*, 30(9): 1105-1112.
8. Chakravarty A. (1985). "Multiproduct purchase scheduling with limited budget and/or group discounts", *Computers and Operations Research*, 12(5): 493-505.
9. Chakravarty A. and Martin G. (1988). "An optimal joint buyer-seller discount pricing model", *Computers and Operations Research*, 15(3): 271-281.
10. Chakravarty, A. (1986). "Quantity discounted inventory replenishments with limited storage space", *INFOR*, 24(1): 12-25.

11. Chang H., Yao J., Ouyang L. (2006). "Fuzzy mixture inventory model involving fuzzy random variable lead time demand and fuzzy total demand", *European Journal of Operational Research*, 169: 65-80.

12. Chen S., Wang C., Arthur R. (1996). "Backorder fuzzy inventory model under function principle", *Information Sciences*, 95(1-2): 71-79.

13. Corbett C. and Groote X. (2000). "A supplier's optimal quantity discount policy under asymmetric information", *Management Science*, 46(3): 444-450.

14. Gen M.,Tsujimura Y., Pa Z. (1997). "An application of fuzzy set theory to inventory control models", *Computers and Industrial Engineering*, 33: 553-556.

15. Goyal S. (1987). "Comment on: A generalized quantity discount pricing model to increase supplier's profits", *Management Science*, 33(12): 1635-1636.

16. Goyal S. (1987). "Determination of a supplier's economic ordering policy", *Journal of the Operational Research Society*, 38(9): 853-857.

17. Hsieh C. (2002). "Optimization of fuzzy production inventory models", *Information Sciences*, 146: 29-40.

18. Hwang H. and Kim K. (1986). "Suppliers discount policy with a single price break point", *Engineering Cost and Production Economics*, 10(3): 279-286.

19. Kim K. and Hwang H. (1988). "Anincremental discount pricing schedule with multiple customers and single price break", *European Journal of Operational Research*, 35(1): 71-79.

20. Kohli R. and Park H. (1989). "A cooperative game theory model of quantity discounts", *Management Science*, 35(6): 693-707.

21. Lam S. and Wong D. (1996). "A fuzzy mathematical model for the joint economic lot size problem with multiple price breaks", *European Journal of Operational Research*, 95(3): 611-622.

22. Lee H. and Rosenblatt M. (1986). "A generalized quantity discount pricing model to increase supplier's profits", *Management Science*, 32(9): 1177-1185.

23. Lee H. and Yao J. (1998). "Economic production quantity for fuzzy demand quantity and fuzzy production quantity", *European Journal of Operational Research*, 109(1): 203-211.

24. Lee H. and Yao J. (1999). "Economic order quantity in fuzzy sense for inventory without backorder model", *Fuzzy Sets and System*, 105(1): 13-31.

25. Lee K. (2005). "First Course on Fuzzy Theory and Application", *Springer-Verlag, Berlin Heidelberg.*

26. Li J. and Liu L. (2006). "Supply chain coordination with quantity discount policy", *International Journal of Production Economics*, 1(89-98).

27. Lin D. and Yao J. (2000). "Fuzzy economic production for production inventory", *Fuzzy Sets and System*, 111(3): 465-495.

28. Mahata G., Goswami A., Gupta D. (2005). "A joint economic-lot-size model for purchaser and vendor in fuzzy sense", *Computers and Mathematics with Applications*, 50(10-12): 1767-1790.

29. Monahan J. (1984). "A quantity discount pricing model to increase vendor profits", *Management Science*, 30(6): 720-726.

30. Munson C. and Rosenblatt M. (1998). "Theories and realities of quantity discounts: An exploratory study", *Production and Operations Management*, 7(4): 352-369.

31. Munson L. and Rosenblatt J. (2001). "Coordinating a three level supply chain with quantity discounts", *IIE Transactions*, 33(4): 371-384.

32. Park K. (1987). "Fuzzy-set theoretic interpretation of economic order quantity", in: *IEEE Transactions on Systems, Man, and Cybernetics SMC-17*, 1082-1084.

33. Parlar M. and Wang Q. (1994). "Discounting decisions in a supplier-buyer relationship with a linear buyer's demand", *IIE Transactions*, 26(2): 34-41.

34. Qin Y., Tang H., Guo C. (2007). "Channel coordination and volume discounts with price-sensitive demand", *International Journal of Production Economics*, 105(1): 43-53.

35. Rosenblatt M. and Lee H. (1985). "Improving profitability with quantity discounts under fixed demand", *IIE Transactions*, 17(4): 388-395.

36. Sarmaha S. P., Acharyaa D., Goyal S. K. (2006). "Buyer vendor coordination models in supply chain management", *European Journal of Operational Research*, 175: 1-15.

37. Shin H. and Benton W. (2006). "A quantity discount approach to supply chain coordination", *European Journal of Operational Research*, Article in press.

38. Sinha S. and Sarmah S. P. (2008). "An application of fuzzy set theory for supply chain coordination", *International Journal of Management Science and Engineering Management*, 3(1): 19-32.

39. Sucky E. (2006). "A bargaining model with asymmetric information for a single supplier-single buyer problem", *European Journal of Operational Research*.

40. Viswanathan S. and Wang Q. (2003). "Discount Pricing decisions in distribution channels with price-sensitive demand", *European Journal of Operational Research*, 149(3): 571-587.

41. Vujosevic M., Petrovic D., Petrovic R. (1996). "EOQ formula when inventory cost is fuzzy", *International Journal of Production Economics*, 45(1-3): 499-504.

42. Weng Z. (1995). "Channel coordination and quantity discounts", *Management Science*, 41(9): 1509-1522.

43. Yang C. and Wee M. (2001). "An arborescent inventory model in a supply chain system", *Production Planning and Control*, 12(8): 728-735.

44. Yao J. and Chiang J. (2003). "Inventory without backorder with fuzzy total cost and fuzzy storing cost de-fuzzified by centroid and signed distance", *European Journal of Operational Research*, 148(2): 401-409.

45. Yao J. and Lee H. (1996). "Fuzzy inventory with backorder for fuzzy order quantity", *Information Science*, 93(3-4):283-319.

46. Yao J. and Su J. (2000). "Fuzzy inventory with backorder for fuzzy total demand based on interval-valued fuzzy set", *European Journal of Operational Research*, 124(2): 390-408.

47. Zadeh L. (1965). "Fuzzy sets", *Information and Control*, 8(3): 338-353.

48. Zhou Y. and Li D. (2007). "Coordinating order quantity decisions in the supply chain contract under random demand", *Applied Mathematical Modelling*, 31(6):1029-1038.

Appendix

Fuzzy set theory

Definition 1. A fuzzy set is a set where the members are allowed to have partial membership and hence the degree of membership varies from 0 to 1. It is expressed as, $A = \left\{\left(x, \mu_{\tilde{A}}(x)\right) \middle| x \in X\right\}$ where X is the universe of discourse and $\mu_{\tilde{A}}(x)$ is the universe of discourse and $\mu_{\tilde{A}}(x) = 0 or 1$ *i.e.*, x is a non-member in A if $\mu_{\tilde{A}}(x) = 0$, and x is a member in A if $\mu_{\tilde{A}}(x) = 1$.

Definition 2. If a fuzzy set A is defined on X, for any $\alpha \in [0,1]$, the α-cuts ${}^{\alpha}A$ is represented by the following crisp set,

$$\text{Strong}\,\alpha-cuts:\ {}^{\alpha+}A=\left\{x\in X\middle|\mu_A(x)>\alpha\right\};\alpha\in[0,1]$$

$$\text{Weak}\ \alpha-cuts:\ {}^{\alpha}A=\left\{x\in X\middle|\mu_A(x)\geq\alpha\right\};\alpha\in[0,1].$$

Fuzzy Arithmetic Operations: We define fuzzy arithmetic operations on fuzzy numbers in terms of the α-cuts. Let, A and B are two fuzzy sets and if '$*$' denotes any of the four basic arithmetic operations $(+,-,*,/)$ then a fuzzy set $Z=(A*B)$ and $Z\in R$, can be defined as, ${}^{\alpha}(A*B)={}^{\alpha}A*{}^{\alpha}B$ such that $\forall\alpha\in(0,1]$.

However, if '$*$' is a division operator, then ${}^{\alpha}(A*B)={}^{\alpha}A*{}^{\alpha}B$, such that $\forall\alpha\in(0,1]$ and $0\notin{}^{\alpha}B$.

Theorem 1. *(First decomposition theorem)*

For every $A\in X$,

$$A=\bigcup_{\alpha\in(0,1]}{}_{\alpha}A,\ where,\ {}_{\alpha}A(x)=\alpha\,{}_{\alpha}A(x)$$

From first decomposition theorem, if $Z=(A*B)$ and $Z\in R$,

$$(A*B)=\bigcup_{\alpha\in(0,1]}{}_{\alpha}(A*B).$$

Since ${}^{\alpha}(A*B)$ is a closed interval for each $\alpha\in(0,1]$ with both A and B fuzzy, $(A*B)$ is also a fuzzy number.

Definition 3. For the de-fuzzification of a fuzzy set to a crisp value, 'centre of gravity' or 'moment method' is a popular and efficient approach. $\tilde{A}(x)$ is converted to a crisp value by the following operation,

$$A=defuzz(\tilde{A})=\frac{\int_R A\mu_{\tilde{A}}(x)dx}{\int_R\mu_{\tilde{A}}(x)dx}$$

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