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A two warehouse inventory model for deteriorating items with time dependent partial backlogging and variable demand dependent on marketing strategy and time

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Abstract

This paper deals with a deterministic inventory model for single deteriorating items with two separate storage facilities (own and rented warehouses) due to limited capacity of the existing storage i.e., own warehouse. The demand rate of this item is dependent on continuous time, selling price of the item and the frequency of advertisement in the popular electronic and print media and also through the sales representatives. The replenishment rate is infinite. The stock is transferred from the rented warehouse to own warehouse in continuous release pattern and the associated transportation cost is taken into account. Shortages, if any, are allowed and partially backlogged with a rate dependent on the duration of waiting time up to the arrival of next lot. The corresponding problem has been formulated as a constrained non-linear mixed integer problem. Finally, the model has been illustrated with a numerical example and to study the effect of changes of different system parameters on initial stock level, maximum shortage level, cycle length, frequency of advertisement along with the maximum profit of the system, sensitivity analyses have been carried out by changing one parameter at a time and keeping the other parameters at their original values.

Keywords: Inventory; deterioration; partially backlogging; time dependent demand; two warehouse model.

1. Introduction

In the present competitive market situation, for promoting a product into the market, the management first attempts to advertise the same in the modern mass/ electronic/ point media. The propaganda and canvassing of a product by time-to-time glamorous advertisements in the well known media such as T.V., Radio, Newspaper, Magazine, etc. and also through the sales representatives have a motivational effect on the public to buy more. As a result, we can conclude that the demand of a product is dependent on the frequency of advertisement of the same. On the other hand, the selling price of an item plays a crucial role for selecting the same for use. Again, the demand of some items, specially seasonal products, increases with time. Hence, it can be concluded that the demand rate of an item is dependent on the frequency of advertisements, selling price and time simultaneously.

According to the existing literature of inventory control theory, most of the inventory models have been developed for single warehouse facility under the basic assumption that the management owns a warehouse with limited capacity. However, in reality, a warehouse is always of limited capacity. On the other hand, when the attractive price discount for bulk purchase is available or the cost procuring goods is higher than the inventory related cost or the demand of items is very high, the management then decides to purchase (or to produce) a large quantity of items at a time. These items can not be stored in the existing warehouse (viz. OW) with limited capacity. Then for storing the excess items, one (sometimes more than one) more warehouse, located away from or nearer to OW, is hired on rental basis. The demand of the customer is met from OW only. Usually, the holding cost in the rented warehouse (viz. RW) will be greater than that in OW. Hence, in order to reduce the holding cost, the items are stored first in OW and only the excess stock is stored in RW. Again, the stocks of RW are emptied first by transferring the stocks from RW to OW in either continuous or bulk release pattern. This system is known as two-warehouse inventory system.

During the last two decades, a number of research papers on two-warehouse inventory system has been published by several researchers. Hartely (1976) first proposed this problem in his book "Operations Research – A Managerial Emphasis". In the formulation, the transportation cost for transferring the items from RW to OW was not considered. After Hartely (1976), Sarma (1983) extended the model under the assumption that the stocks of RW are transferred from RW to OW in a bulk release fashion with fixed transportation cost per unit. Dave (1988) discussed the two-warehouse inventory models for finite and infinite rate of replenishment rectifying the errors of the model developed by Sarma (1983) and gave analytical solution of each model. Further, Goswami and Chaudhuri (1992) developed the

models with and without shortages for linearly time dependent demand. In their formulation, stocks of RW are transferred to OW in equal time interval. Correcting and modifying the assumptions of Goswami and Chaudhuri (1992), Bhunia and Maiti (1994) investigated the same model and graphically presented the sensitivity analysis on the optimal average cost and the cycle length for the variations on the location and shape parameters of demand. The same type of model was developed by Kar et al.(2001), Zhou and Yang(2003) and Mondal et al.(2007) for different types of demand.

In reality, there are so many physical goods which deteriorate during the stock-in period due to different factors, like, dryness, damage, spoilage, vaporization, etc. This deterioration effect depends on the preserving facilities of the warehouses. Hence, this effect can not be ignored in the studies of inventory problems. Considering this effect in both warehouses with different preserving facilities, Sarma (1987) first developed a two-warehouse inventory model with infinite replenishment and completely backlogged shortages. Pakkala and Achary (1992) then extended Sarma's (1987) model for finite replenishment rate. These models were developed for prescribed scheduling period (cycle length), uniform demand and stocks of RW transferred to OW in continuous release fashion ignoring the transportation cost for this purpose. Benkherouf (1997) presented a two-warehouse model for deteriorating items with the general form of time dependent demand under continuous release fashion. Bhunia and Maiti (1997) discussed the same type of problem considering linearly (increasing) time dependent demand with completely backlogged shortages. This model was developed for infinite time horizon with the repetition of entire cycle with the changed value of the location parameter of the time dependent demand. Recently, several researchers developed this type of model considering finite time horizon. All these models mentioned earlier were discussed only for non-deteriorating items. Lee and Ma (2000) developed a no-shortage inventory model for perishable items with free form of time dependent demand and fixed planning horizon. In their models, some of the cycles are of single warehouse system and the remaining cycles are of two-warehouse system. Kar et al. (2001) discussed two storage inventory problems for non-perishable items with linear trend in demand over a fixed planning horizon considering lot-size dependent replenishment cost (ordering cost). On the other hand, considering two-storage facilities, Yang (2004) developed two inventory models for deteriorating items with uniform demand rate and completely backlogged shortages under inflation. Recently, Yang (2006) extended the models introduced in Yang (2004) by incorporating the partially backlogged shortages. In the year 2007, Chung and Huang (2007) proposed two-warehouse inventory model for

deteriorating items. However, all these models were developed based on an impractical assumption that the rented warehouse has unlimited capacity.

In this paper, we have developed an inventory model for single deteriorating items with two warehouses (one is OW and other is RW) having limited capacities by removing the impractical assumption regarding the storage capacity of rented warehouse. Due to different preserving facilities and storage environments, separate inventory holding costs and deterioration rates are considered in the different warehouses. The demand rate is assumed to be variable and it is dependent on continuous time, selling price and the frequency of the advertisement. The stocks of RW are transferred to OW in continuous release pattern. Shortages, if any, are allowed and partially backlogged with a rate dependent on the duration of waiting time up to the arrival of next lot. The corresponding problem is formulated as a constrained non-linear mixed integer programming problem with three decision variables having one as integer and other two as non-integer (floating point) variables. Now, to illustrate the results of the system, a numerical example has been solved. Finally, to study the effect of under and over estimations of different parameters, like demand, deterioration and backlogging rate, on initial and reorder point stock levels, cycle length and frequency of advertisement along with the maximum profit of the system, sensitivity analyses have been performed.

2. Assumptions and Notations

The following assumptions and notations have been used in developing the model.

(i) Replenishment rate is infinite and lead-time is constant.

(ii) The entire lot size is delivered in one batch.

(iii) The inventory planning horizon is infinite and the inventory system involves only one item.

(iv) Deterioration is considered only after the inventory stored in the warehouse. There is neither repair nor replacement of the deteriorated units during the inventory cycle.

(v) T be the cycle length of the system

(vi) Shortages, if any, are allowed and partially backlogged. During the stock-out period, the backlogging rate is dependent on the length of the waiting time up to the arrival of fresh lot. Considering this situation, this rate is defined as $\left[1+\delta\left(T-t\right)\right]^{-1}$, where $\delta > 0$, T be the cycle length of the inventory system and t, the instantaneous time during stock-out period.

(vii) q(t) be the inventory level at any time t.

(viii) S be the highest stock level at the beginning of the cycle.

(ix) W_1, W_2 be the storage capacities of OW and RW.

(x) α, β be the deterioration rates in OW and RW respectively and $0 < \alpha, \beta << 1$.

(xi) R be the shortage level at reorder point.

(xii) C_4 be the transportation cost per unit for transferring the items from RW to OW.

(xiii) A_1 be the replenishment cost(ordering cost) for replenishing the item in OW.

(xiv) A_2 be the additional cost for replenishing the excess items in RW.

(xv) m(> 1) be the mark-up rate.

(xvi) C_3 be the purchase cost per unit quantity of item.

(xvii) p be the selling price per unit of item which is of the form $p = mC_3$.

(xviii) A be the frequency of advertisement.

(xix) The demand rate D(A, p, t) is dependent on continuous time (t), selling price(p) of an item and the frequency of advertisement(A). We assume it as follows:

$D\left(A, p, t\right) = A^{\gamma}\left(a - bp + ct\right)$ where $a, b, c, \gamma \geq 0$ and A is an integer.

(xx) C_5 be the advertisement cost per advertisement.

(xxi) C_1, C_1' $\left(C_1' > C_1\right)$ be the holding costs per unit per unit time at OW and RW respectively.

(xxii) C_2 be the shortage cost per unit per unit time.

(xxiii) The capacity of a transport vehicle is k units.

(xxiv) L be the distance between the show-room / shop and the source of the items / commodities from where items / commodities to be transported.

(xxv) C_t be the transportation cost for full load of the transport vehicle and C_{tF} be the transportation cost per unit item.

(xxvi) U be the upper break point. Some quantities less than k but above U, the transportation cost for the whole quantity is C_t.

Hence $U = \left[C_t / C_{tF}\right] \left(< k\right)$ where $\left[C_t / C_{tF}\right]$ represents the greatest integer value which is less than or equal to C_t / C_{tF}.

3. The Mathematical Model

Initially, an enterprise purchases (S + R) units of the item. Then after fulfilling the backorder quantities, the on-hand inventory level is S units of which W_1 units are stored in OW and the remaining amount (S-W_1) units in RW. During the time interval $0 \le t \le t_1$, the inventory (S – W_1) units in RW decreases due to meet up the customers' demand and deterioration effect of the item and it varnishes at $t = t_1$. In OW, the inventory level W_1 decreases during $0 \le t \le t_1$ due to deterioration only and during $t_1 < t \le t_2$ due to both the demand and deterioration. At time $t = t_2$, the inventory level in OW reaches to zero. There after the shortages are allowed to occur during the time interval $t_2 < t \le T$ with a rate $\left[1 + \delta\left(T - t\right)\right]^{-1}, (\delta > 0)$. At time $t = T$, the maximum shortage level is R. This entire cycle then repeats itself after the cycle length T with the changed demand as it is time dependent. Our objective is to determine the optimal values of A, t_1, t_2 and T such that the average profit of the system is maximized and also to obtain the corresponding values of S and R.

The stock depletion at RW during $0 \le t \le t_1$ is mainly due to demand and partly due to deterioration of the items which are disposed continuously as they come to the selling point. Hence the inventory level $Q_r\left(t\right)$ at RW satisfies the following differential equation:

$$Q_r'\left(t\right) + \beta Q_r\left(t\right) = -A^{\gamma}\left(a - bp + ct\right), \qquad 0 \le t \le t_1 \qquad ...(1)$$

subject to the condition that

$$Q_r\left(t\right) = 0 \quad \text{at} \quad t = t_1. \qquad ...(2)$$

$$\text{Again, } Q_r\left(t\right) = S - W_1 \quad \text{at} \quad t = 0. \qquad ...(3)$$

With the help of (2), the solution of (1) is given by

$$Q_r\left(t\right) = \begin{bmatrix} \left\{\beta A^{\gamma}\left(a - bp + ct_1\right) - A^{\gamma}c\right\}\exp\left\{\beta\left(t_1 - t\right)\right\} \\ -\left\{\beta A^{\gamma}\left(a - bp + ct\right) - A^{\gamma}c\right\} \end{bmatrix} / \beta^2, \; 0 \le t \le t_1 \qquad ...(4)$$

From (3) and (4), we have

$$S = W_1 + \left[\left\{\beta A^{\gamma}\left(a - bp + ct_1\right) - A^{\gamma}c\right\}\exp\left(\beta t_1\right) - \left\{\beta A^{\gamma}\left(a - bp\right) - A^{\gamma}c\right\}\right] / \beta^2 \quad ...(5)$$

Again the inventory level $Q_0(t)$ at OW during the time interval $0 \le t \le T$ satisfies the differential equations as follows:

$$Q_0'(t) + \alpha Q_0(t) = 0, \quad 0 \le t \le t_1 \quad ...(6)$$

$$Q_0'(t) + \alpha Q_0(t) = -A^{\gamma}\left(a - bp + ct\right), \quad t_1 < t \le t_2 \quad ...(7)$$

$$Q_0'(t) = -\frac{A^{\gamma}\left(a - bp + ct\right)}{1 + \delta\left(T - t\right)}, \quad t_2 < t \le T \quad ...(8)$$

subject to the conditions

$$Q_0(t) = W_1 \quad \text{at} \quad t = 0 \quad ...(9)$$

$$Q_0(t) = 0 \quad \text{at} \quad t = t_2 \quad ...(10)$$

and $Q_0(t) = -R \quad \text{at} \quad t = T.$ ...(11)

Again $Q_0(t)$ is continuous at $t = t_1$ and $t = t_2$.

Using the conditions (9) – (11), the solutions of the differential equations (6) – (8) are given by

$$Q_0(t) = W_1 \exp\left(-\alpha t\right), \quad 0 \le t \le t_1 \quad ...(12)$$

$$= \begin{bmatrix}\left\{\alpha A^{\gamma}\left(a - bp + ct_2\right) - A^{\gamma}c\right\}\exp\left\{\alpha\left(t_2 - t\right)\right\} \\ -\left\{\alpha A^{\gamma}\left(a - bp + ct\right) - A^{\gamma}c\right\}\end{bmatrix} / \alpha^2, \ t_1 < t \le t_2 \quad ...(13)$$

$$= A^{\gamma}\left[\left\{\left(a - bp\right) + c\left(T + \frac{1}{\delta}\right)\right\}\log\left|1 + \delta\left(T - t\right)\right| - c\left(T - t\right)\right] / \delta - R, \ t_2 < t \le T \quad ...(14)$$

Now using the continuity condition of $Q_0(t)$ at time $t = t_1$, we have

$$W_1 \exp\left(-\alpha t_1\right) = \begin{bmatrix} \left\{\alpha A^{\gamma}\left(a - bp + ct_2\right) - A^{\gamma}c\right\}\exp\left\{\alpha\left(t_2 - t_1\right)\right\} \\ -\left\{\alpha A^{\gamma}\left(a - bp + ct_1\right) - A^{\gamma}c\right\} \end{bmatrix} / \alpha^2$$

i.e., $$\begin{matrix} \left\{\alpha A^{\gamma}\left(a - bp + ct_2\right) - A^{\gamma}c\right\}\exp\left(\alpha t_2\right) \\ -\left\{\alpha A^{\gamma}\left(a - bp + ct_1\right) - A^{\gamma}c\right\}\exp\left(\alpha t_1\right) - \alpha^2 W_1 = 0 \end{matrix} \quad ...(15)$$

Again, from the continuity of $Q_0\left(t\right)$ at $t = t_2$, we have

$$R = A^{\gamma}\left[\left\{a - bp + c\left(T + \frac{1}{\delta}\right)\right\}\log\left|1 + \delta\left(T - t_2\right)\right| - c\left(T - t_2\right)\right] / \delta \quad ...(16)$$

Now the total deteriorated units in RW and OW over the stock-in period are given by

$D_1' = \beta\int_0^{t_1} Q_r\left(t\right)dt$ and $D_2' = \alpha\int_0^{t_2} Q_0\left(t\right)dt$ respectively.

Hence we can write

$$\int_0^{t_1} Q_r\left(t\right)dt = D_1' / \beta \quad \text{and} \quad \int_0^{t_2} Q_0\left(t\right)dt = D_2' / \alpha \quad ...(17)$$

Again, $$D_1' = S - W_1 - \int_0^{t_1} A^{\gamma}\left(a - bp + ct\right)dt \quad ...(18)$$

and $$D_2' = W_1 - \int_{t_1}^{t_2} A^{\gamma}\left(a - bp + ct\right)dt \quad ...(19)$$

Hence the total holding cost C_{hold} over the entire cycle is given by

$$C_{hold} = C_1'\int_0^{t_1} Q_r\left(t\right)dt + C_1\int_0^{t_2} Q_0\left(t\right)dt$$

$$= C_1'\left[S - W_1 - \int_0^{t_1} A^{\gamma}\left(a - bp + ct\right)dt\right] / \beta + C_1\left[W_1 - \int_{t_1}^{t_2} A^{\gamma}\left(a - bp + ct\right)dt\right] / \alpha$$

...(20)

Again, the total shortage cost C_{sho} over the entire cycle is given by

$$C_{sho} = C_2 \int_{t_2}^{T} \left[-Q_0(t)\right] dt = -C_2 \left[A^{\gamma} \left\{ \left(a - bp + c\left(T + \frac{1}{\delta} \right) \right) \left(1 + \delta\left(T - t_2 \right) \right) \right.\right.$$

$$\left.\left. \left(\log\left| 1 + \delta\left(T - t_2 \right) \right| / \delta - \left(T - t_2 \right) \right) - c\left(T - t_2 \right)^2 / 2 \right\} + R\left(T - t_2 \right) \right] \quad ...(21)$$

The transportation cost C_{tran1} for transferring the items from RW to OW is given by

$$C_{tran1} = C_4 \int_0^{t_1} A^{\gamma} \left(a - bp + ct \right) dt = C_4 A^{\gamma} \left\{ \left(a - bp \right) t_1 + \frac{c}{2} t_1^2 \right\} \quad ...(22)$$

Again, the transportation cost C_{tran2} for replenishment of items is given by

$$C_{tran2} = nC_t + \left(S + R - nk \right) C_{tF} \quad \text{when } nk < S + R \le nk + U$$

$$= \left(n + 1 \right) C_t \quad \text{when } nk + U < S + R \le \left(n + 1 \right) k$$

Hence the total transportation cost C_{tran} is given by

$$C_{tran} = C_{tran1} + C_{tran2}$$

During the entire inventory cycle, the total cost TC of the system is given by

TC = <ordering cost> + <advertisement cost> + <purchase cost> + <holding cost> + <shortage cost> + <transportation cost>

i.e. $TC = A_1 + A_2 + C_5 A + C_3 \left(S + R \right) + C_{hold} + C_{short} + C_{tran}$ ...(23)

Therefore the net profit (X) of the system is the difference between the sales revenue and the total cost of the system, i.e.

$$X = p \left[A^{\gamma} \left\{ \left(a - bp \right) t_2 + \frac{c}{2} t_2^2 \right\} + R \right] - C_3 \left(S + R \right) - \left(A_1 + A_2 \right)$$

$$- C_5 A - C_{hold} - C_{short} - C_{tran} \quad ...(24)$$

Hence, the profit function Z (average profit per unit time) for the entire cycle of the system is given by

$$Z\left(m, n, A, t_1, t_2, T \right) = \frac{X}{T}$$

i.e. $Z\left(m,n,A,\ t_1,t_2,T\right)=\left[p\left\{A^{\gamma}\left\{\left(a-bp\right)t_2+\frac{c}{2}t_2^2\right\}+R\right\}-C_3\left(S+R\right)-\left(A_1+A_2\right)\right.$

$\left.-C_5A-C_{hold}-C_{short}-C_{tran}\right]/T$ ...(25)

where m, t_1,t_2,T are four continuous variables and n, A are two integer variables. The above function $Z\left(m,n,A,\ t_1,t_2,T\right)$ is a highly nonlinear function due to presence of exponential as well as logarithmic functions. Here Z is an increasing function with respect to m. It can be shown graphically for the prescribed values of the parameters but not analytically. Hence our problem is to determine the optimal values of n, A, T and S for the fixed value of the mark-up rate m by solving the following mixed integer nonlinear optimization problem.

Maximize $Z\left(n,A,t_1,t_2,T\right)$

subject to the constraints

$$\left\{\alpha A^{\gamma}\left(a-bp+ct_2\right)-A^{\gamma}c\right\}\exp\left(\alpha t_2\right)-\left\{\alpha A^{\gamma}\left(a-bp+ct_1\right)-A^{\gamma}c\right\}\exp\left(\alpha t_1\right)-\alpha^2W_1=0$$

$0<t_1<t_2<T$ and $W_1<S\le W_1+W_2$

where $C_{tran2}=nC_t+\left(S+R-nk\right)C_{tF}$ when $nk<S+R\le nk+U$

$=\left(n+1\right)C_t$ when $nk+U<S+R\le\left(n+1\right)k$

The above problem can be solved by using the well known generalized reduced gradient (GRG) method and the following algorithm.

Algorithm

Step-1: Input all the parameters.

Step-2: Calculate the value of U.

Step-3: Using GRG method, solve the above problem by taking the transportation cost for first situation $(nk<S+R\le nk+U)$.

Step-4: If (S+R) lies in the interval $(nk<S+R\le nk+U)$, then this is the optimal policy and go to Step-7. Otherwise, go to Step-5.

Step-5: Using GRG method, solve the above problem by taking the transportation cost for second situation $(nk+U<S+R\le(n+1)k)$.

Step-6: If (S+R) lies in the interval $(nk + U < S + R \leq (n+1)k)$, then this is the optimal policy.

Step-7: Stop.

4. Numerical Illustrations

To illustrate the developed model, a numerical example with the following values of different parameters has been considered.

Let C_1 = \$1.00 per unit per unit time, C_1' = \$0.50 per unit per unit time, A_1 = \$150.00 per order, A_2 = \$50.00 per order, C_2 = \$15.00 per unit per unit time, C_3 = \$25.00 per unit, C_4 = \$0.50 per unit, C_{tF} = \$1.25 per unit, C_5 = \$50.00 per advertisement, C_t = \$100.00 per load, S_0 = 50, S_1 = 250, a = 250, b = 5, c = 5, $\theta = 0.1$, γ = 0.1, δ = 1.5, W_1 = 150, W_2 =100, k = 100 unit, β =0.07, α = 0.1, L=50 km.

The parameter values considered here are realistic, though these values are not taken from any case study of an existing inventory system.

To show the profit function as an increasing function with respect to the mark-up rate m, first of all, we have computed the profit function for different values of m and drawn the graph (see Fig.-1). From Fig.-1, it is evident that the profit Z is an increasing function of m.

According to the proposed algorithm, the optimal solution has been obtained with the help of GRG method for different values of m. The optimum values of n, A, t_1, t_2,T, S and R along with maximum average profit are displayed in Table 1.

Table 1: Optimal solution for different values of mark-up rate m

m	A	S	R	t_1	t_2	T	Z
1.25	1	153.6615	26.3385	0.03896	1.46354	1.77925	119.4907
1.27	1	154.4600	25.5400	0.04872	1.50495	1.81765	156.9375
1.30	2	161.7843	25.8259	0.12466	1.5284	1.83341	211.8844
1.32	2	160.8528	24.93338	0.118225	1.55853	1.85977	246.311
1.35	2	159.3756	23.69333	0.106906	1.60585	1.90258	293.4273

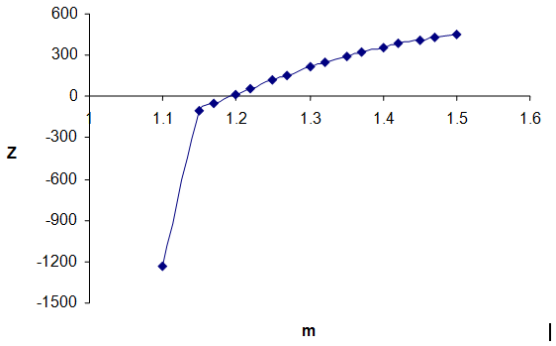


Fig.-1 : Mark-up rate (m) vs. Average profit (Z)

5. Sensitivity Analysis

The earlier numerical example is used to study the effect of under or over estimation of system parameters on the optimal values of the initial stock level, maximum shortage level, cycle length, frequency of advertisement along with the maximum profit of the system. The percentage changes in the above mentioned optimal values are taken as measures of sensitivity. The analysis is carried out by changing (increasing and decreasing) the parameters by -20% to +20%. The results are obtained by changing one parameter at a time and keeping the other parameters at their original values. The results of these analysis are given in Tables 2 which are self- explainatory.

Table 2 : Sensitivity analysis with respect to different parameters

Parameter	% changes of parameters	% changes in Z^*	A^*	% changes in				
				S^*	R^*	t_1^*	t_2^*	T^*
C_1	– 20	0.04	2	0.75	0.06	10.27	0.67	0.56
	– 10	0.02	2	0.36	0.03	4.90	0.32	0.27
	10	-0.02	2	-0.33	-0.03	-4.41	-0.29	-0.25
	20	-0.03	2	-0.63	-0.05	-8.51	-0.56	-0.47
C_1'	– 20	3.16	2	2.05	-1.26	27.85	1.83	1.24
	– 10	29.89	2	1.03	-0.63	14.13	0.92	0.63
	10	-1.55	2	-1.05	0.62	-14.29	-0.94	-0.32
	20	-3.09	2	-2.11	1.23	-28.89	-1.9	-1.3
C_2	– 20	3.26	2	-1.35	14.71	-18.35	-1.21	2.00
	– 10	1.52	2	-0.63	6.85	-8.59	-0.56	0.91
	10	-1.34	2	0.56	-6.02	7.62	0.50	-0.77
	20	-2.52	2	1.05	-11.36	14.37	0.89	-1.43
C_3	– 20	3.94	1	7.88	-1.40	62.28	-13.40	-15.08
	– 10	6.88	2	7.2	2.86	67.09	-8.10	-8.99
	10	-16.1	2	-4.6	-0.67	-54.57	13.9	15.26
	20	-42.51	2	-5.21	3.16	-54.49	38.77	42.89

C_5	– 20	5.21	2	-2.67	-2.71	-36.44	-2.39	-2.49
	– 10	2.59	2	-1.33	-1.35	-18.06	-1.20	-1.24
	10	-2.7	1	-3.88	-5.16	-49.76	2.68	2.51
	20	-3.95	1	-3.92	-4.94	-50.24	2.64	2.53
W_1	– 20	-4.68	1	-3.73	-6.08	219.66	3.36	2.53
	– 10	-3.16	1	-3.79	-5.71	86.60	2.80	2.49
	10	1.72	2	1.99	-0.63	-100.00	1.75	1.31
	20	1.38	2	11.26	-22.56	-100.00	9.91	3.86
A_1	– 20	7.87	2	-4.03	-4.08	-55.15	-3.69	-3.76
	– 10	3.9	2	-1.99	-2.03	-27.21	-1.97	-1.86
	10	-3.83	2	1.95	2.00	26.57	1.75	1.82
	20	-7.59	2	3.87	3.98	52.57	3.45	3.61
A_2	– 20	2.59	2	-1.33	-1.35	-18.06	-1.18	-1.24
	– 10	1.29	2	-0.66	-0.67	-8.99	-0.59	-0.62
	10	-1.28	2	0.66	0.67	8.91	0.59	0.61
	20	-2.56	2	1.31	1.34	17.82	1.17	1.22
a	– 20	-55.91	1	-4.39	-1.98	-44.22	29.35	28.29
	– 10	59.35	2	6.84	5.15	50.40	-14.35	-15.28
	10	59.35	2	6.84	5.15	50.40	-14.35	-15.28
	20	119.93	2	6.62	6.50	21.51	-30.09	-29.55
b	– 20	77.46	2	6.78	5.50	40.61	-19.49	-20.12
	– 10	38.33	2	6.89	4.81	63.48	-8.34	-8.87
	10	-36.61	2	-4.29	-2.57	-49.36	14.22	15.04
	20	-72.38	1	-4.18	-3.28	-27.29	48.41	50.61
c	– 20	-1.84	2	-1.17	-1.97	-15.97	-0.29	-0.28
	– 10	-0.92	2	-0.59	-0.99	-7.95	-0.14	-0.14
	10	0.92	2	0.59	0.99	8.03	0.14	0.14
	20	1.85	2	1.18	1.98	16.05	-0.03	0.28
α	– 20	17.86	2	8.85	-7.47	119.5	9.49	6.32
	– 10	8.70	2	4.89	-3.45	66.29	5.12	3.51
	10	-8.19	2	-5.23	3.30	-57.46	-5.34	-3.7
	20	-16.00	2	-6.19	9.32	-84.9	-6.8	-3.68
δ	– 20	1.09	2	-0.47	6.07	-6.42	-0.43	0.14
	– 10	0.53	2	-0.23	2.94	-3.13	-0.21	0.07
	10	-0.5	2	0.22	-2.76	3.05	0.2	-0.07
	20	-0.98	2	0.43	-5.35	-5.3	0.38	-0.14
γ	– 20	-1.44	1	-3.87	-5.37	-49.20	2.70	2.49
	– 10	-1.44	1	-3.85	-5.37	-49.20	2.70	2.49
	10	1.47	2	0.39	0.30	4.65	-0.27	-0.29
	20	2.95	2	0.79	0.61	9.23	-0.53	-0.59

6. Concluding Remarks

In this paper, an attempt has been made to incorporate the different preservation facilities in own and rented warehouses for deteriorating items with variable demand dependent on marketing variables like, selling price of the item and frequency of advertisement in popular media and continuous time. It is also assumed that the preservation facility in RW is better than in OW.

As the rate of demand is assumed to be time dependent for the fixed values of selling price of the item and number of advertisement, so the entire cycle over the first period can not repeat after the cycle length T. It can subsequently be repeated with the changed value of the constant 'a' computed from a + cT where T is associated value of the previous cycle.

In this paper, the demand rate is taken as $D\left(A,p,t\right)=A^{\gamma}\left(a-bp+ct\right)$. It is well known that $D\left(A,p,t\right)\propto\left(a-bp+ct\right)$ for fixed A. But, why should we take $D\left(A,p,t\right)\propto A^{\gamma}$ for fixed values of p and t ? Generally, the demand varies due to the advertisement in the well known media such as Radio, T.V., Newspaper, Magazine, Cinema, etc. The demand of items increases with the increase of frequency of advertisement and is directly proportional to the number of advertisement. Hence, we can take $D\left(A,p,t\right)\propto A^{\gamma}$ for fixed p and t.

The present model is also applicable to the problems where the selling prices of the items as well as the advertisement of items affect the demand. It is applicable for seasonal and fashionable goods.

The two-warehouse model can be applied to many practical situations, due to introduction of open market policy and also to occupy the maximum possible market for highly business competition. For this reason, in order to attract more customers, a departmental store is forced to provide a better purchasing – environment to the customers such as well decorated show-room with modern light and electronic arrangements and enough free space for choosing items, etc. Again, due to the expanding market situation, there is a crisis of space in the market places specially in the super market, corporation market, etc. As a result, the management of the departmental store is bounded to hire a separate warehouse on rental basis at a distance place for storing of excess items. Hence, from the economical point of view, sometimes two-storage system is more profitable than the single storage system.

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