

An economic production lot-sizing model for deteriorating items with partial backlogging under the effect of learning

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ABSTRACT

This paper In this article we have developed an economic production lot-sizing model in infinite horizon. Deterioration is taken into consideration. Shortages is allowed and partially backlogged and the backlogging rate is taken to be waiting time dependent. To make the system more realistic, the effect of learning is also considered. A mathematical formulation has been done. Theoretical results are illustrated with a numerical example. Sensitivity analysis is also given.

Keywords: Deterioration, backlogging , learning effect, Shortages.

1. Introduction

In the past few decades, various articles dealing with production lot size under the effect of learning have appeared on various platforms. But whenever we talk about inventory, deterioration or decaying of holding the stock cannot be disregarded. Deterioration is defined as the spoilage or damage of the inventory that cannot be served to the customer, for example, in the storage of crockery, sometimes it is broken or scratched by mishandling that is the damage and loss of the original worth of the product that resulted in the decreasing utility from the new one. Certain products such as dairy products, vegetable, blood, electronic goods, petrol etc, are the perishable items and their utility decreases under deterioration due to the holding. For inventory control problems, the inventory models for the decaying items have been continually adapted so as to build up more realistic assumptions of the real environment. Different types of inventory models for perishable items were discussed by numerous scholars. The study of deterioration in inventory control system was initiated by the Ghare and Schrader (1963). Zangwill (1966) presented an EPQ

model with allowable shortages. Thereafter Rosenberg (1979) investigated an EOQ model with partial backlogging. Some other researchers developed their models along the same line of research such as Mak (1987) and Wee (1993) using different assumptions. Abad (1996) studied a generalized model of dynamic pricing for deteriorated products. The shortage was permitted which is partially backlogged however the demand was taken to be time dependent. Papachristos and Skouri (2000) presented a lot size model on a finite planning horizon. They developed the model for perishable items with time-varying rates. The problem of formation of the optimal worth and economic quantity for a reseller was evaluated by Abad (2001). The selling price was taken to be constant. Teng et al. (2002) built up a stock model to find the optimal replenishment policy for decaying items. They presented the model with time-dependent demand and the backorders along with the lost sales. An EOQ model with deteriorating items and partial backlogging was proposed by Skouri and Papachristos (2003-b). The backlogging rate was considered as to be a component of the holding up time subordinate. Zhou et al. (2004) established a classical EOQ inventory model with time-varying demand. Shortages were permitted and the partially accumulated, the backlogging rate was considered as waiting time dependent while the replenishment cost was taken to be the function of the order quantity. Chu and Chung (2004) presented have done the same kind of work. Ghosh and Chaudhuri (2005) proposed an economic order quantity model demonstrate with time subordinate decaying and demand rate on a finite planning horizon. Shortages were accumulated and the backlogging rate was waiting time dependent. A mathematical formulation was done to solve the problem and the model was discussed with their previous models and the results were illustrated with a numerical example. Chang et al. (2006-a) then conclude in their model that stocking up the level of inventory is somewhere profitable, and to demonstrate their results, an algorithm was provided by them to find the optimal replenishment policy. In a subsequent study, Chang et al. (2006-b) gave a great deal measure model to a purchaser to set up its ideal offering value, number of conveyances an ideal time with incomplete accumulating. From there on Dye et al. (2007) recommended a deterministic stock model for rotting items with value subordinate request, where the request is taken to be the capacity of cost while crumbling was reliant on time. Deficiencies were permitted and the multiplying rate was considered as the negative exponential capacity of the holding up time. Thangam and Uthayakumar (2008) set up a two-echelon store network stock model with fractional multiplying and the request was approximated with the Poisson circulation. Palanivel and Uthyakumar(2014) developed an economic production quantity (EPQ) model under the effect of inflation and time value of money. The rate of replenishment was considered a variable and the generalized unit production cost function with deterioration was formulated by incorporating several factors, such as raw material, labour, advertisements and other factors of the manufacturing system. Shortages were allowed and partially backlogged.

In this paper, a production stock model for perishable things with deterministic demand has been built up. The demand rate is considered as an element of offering cost. Deteriorating rate is thought to be time subordinate. Backlogging is permitted and partially accumulated. The rate of backlogging units is taken to hold up time subordinate and the production rate depends on time.

2. Assumptions and Notations

Assumptions:

1. The effect of learning is taken into consideration.
2. Items are assumed to be deteriorating in nature and the rate of deterioration is considered as time-varying.
3. The production rate is assumed to be demand dependent.
4. Demand rate is taken to be price dependent $D(p) = \frac{\gamma}{p^\delta}$
5. Shortages are allowed and partially backlogged and backlogging rate is defined as $\Omega(\tau) = k_0 e^{-k_1 \tau}$, where τ is the waiting time and k_0 and k_1 are constant ($k_0 < 1, k_1 \geq 0$).

Notations:

$Q(t)$	Inventory level at any time t
$P = \alpha D$	Production rate
α	Constant ($\alpha > 1$)
$D(p)$	Demand rate

θ	Deterioration rate ($\theta \llll 1$)
p	Selling price.
$(C_p + \frac{C_0}{n^\phi})$	Production cost with learning effect where $\phi > 0$ is the learning coefficient
ξ	Set up cost
h	Storage cost per unit time per unit
γ, δ, ϕ	Arbitrary Constants
q	Total produced items

3. Mathematical Model

As the conduct of stock level delineated in fig. 1., the stock level of the manufacturer is increasing due to the production up to the time $t = \beta$. After that, his inventory is declining because of the consolidated impact of demand and decay. At the time $t = T$ the stock level ends up noticeably zero. Thereafter shortage is partially backlogged up to the time $t = T + \psi$. At the same time production starts to fulfill the unsatisfied demand.

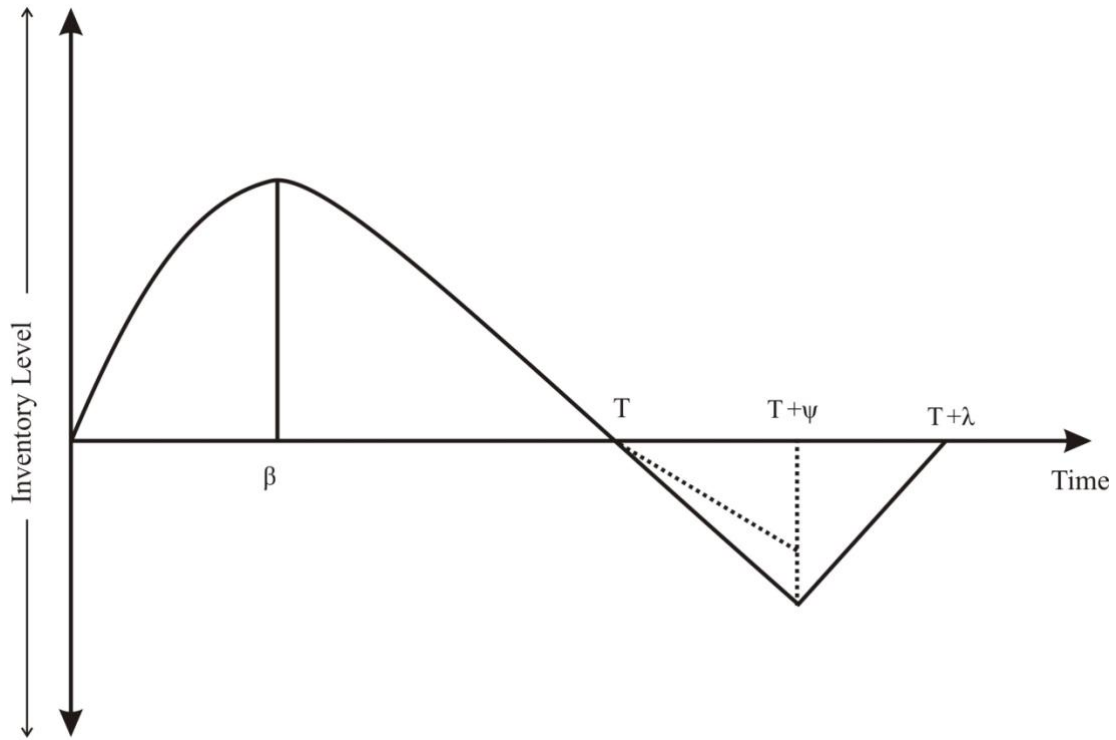


Fig. 1. Behavior of stock level in the production system

The governing differential equations of the model are given by:

$$\frac{dQ(t)}{dt} + \theta t Q(t) = P - D(p), \text{ with } Q(0) = 0, \text{ where } 0 \leq t \leq \beta \quad (1)$$

$$\frac{dQ(t)}{dt} + \theta t Q(t) = -D(p), \text{ with } Q(T) = 0, \text{ where } \beta \leq t \leq T \quad (2)$$

$$\frac{dQ(t)}{dt} = -D(p)\Omega(\tau), \text{ with } Q(T) = 0, \text{ where } T \leq t \leq T + \psi \quad (3)$$

$$\frac{dQ(t)}{dt} = P - D(p)\Omega(\tau), \text{ with } Q(T + \lambda) = 0, \text{ where } T + \Psi \leq t \leq T + \lambda \quad (4)$$

Solution of equation of the first two equations are given as

$$Q(t) = (\alpha - 1) \frac{\gamma}{p^\delta} \left(t + \frac{\theta t^3}{6} \right) \text{Exp} \left[\frac{-\theta t^2}{2} \right], \quad 0 \leq t \leq \beta \quad (5)$$

$$Q(t) = \frac{\gamma}{p^\delta} \left[(T - t) + \frac{\theta}{6} (T^3 - t^3) \right] \text{Exp} \left[\frac{-\theta t^2}{2} \right], \quad \beta \leq t \leq T \quad (6)$$

By the continuity at time $t = \beta$, we get

$$\begin{aligned} Q(\beta) &= \frac{\gamma}{p^\delta} \left\{ (T - \beta) + \frac{\theta}{6} (T^3 - \beta^3) \right\} \text{Exp} \left\{ \frac{-\theta \beta^2}{2} \right\} \\ &= (\alpha - 1) \frac{\gamma}{p^\delta} \left(\beta + \frac{\theta \beta^3}{6} \right) \text{Exp} \left\{ \frac{-\theta \beta^2}{2} \right\} \\ \Rightarrow (T - \beta) + \frac{\theta}{6} (T^3 - \beta^3) &= (\alpha - 1) \left(\beta + \frac{\theta \beta^3}{6} \right) \end{aligned} \quad (7)$$

As depicted in fig.1., in the absence of inventory when production is not occurring, that is if $T < t < T + \psi$, the waiting time would be $T + \psi - t - \frac{Q(t)}{P}$. But after that when production starts, i.e., $T + \psi < t < T + \lambda$ the waiting time is formulated as $\tau = -Q(t) / P$. So we have:

$$\frac{dQ(t)}{dt} = \frac{\gamma}{p^\delta} \Omega \left\{ t - Q(t) / P - (T + \psi) \right\}, \quad T \leq t \leq T + \psi$$

Thus the solution of equation (3) is as follows

$$Q(t) = \frac{P}{k_1} \left\{ \log [P] - \log \left[Dk_0 e^{k_1(t-T-\psi)} + P - Dk_0 e^{-k_1\psi} \right] \right\} \quad (8)$$

Similarly, if $T + \psi < t < T + \lambda$,

$$\frac{dQ(t)}{dt} = -\frac{\gamma}{p^\delta} \Omega \left(-\frac{Q(t)}{P} \right) + P, \quad (T + \psi) \leq t \leq T + \lambda$$

$$\frac{dQ(t)}{dt} = P - \frac{\gamma}{p^\delta} k_0 \text{Exp} \left\{ \frac{k_1 Q(t)}{P} \right\} \quad (9)$$

From equation (8), we get

$$Q(T + \psi) = \frac{P}{k_1} \left\{ \log [P] - \log \left[\frac{\gamma}{p^\delta} k_0 + P - \frac{\gamma}{p^\delta} k_0 e^{-k_1 \psi} \right] \right\} \quad (10)$$

From equation (10) the solution of equation (9) is given by

$$Q(t) = \frac{P}{k_1} \left\{ \log [P] - \log \left[P e^{-k_1(t-T-\psi)} - \frac{\gamma}{p^\delta} k_0 e^{-k_1(t-T)} + \frac{\gamma}{p^\delta} k_0 \right] \right\},$$

$$\text{if } T + \psi \leq t \leq T + \lambda \quad (11)$$

Using the boundary condition $Q(T + \lambda) = 0$, in equation (11) we have

$$\begin{aligned}
Q(t + \lambda) &= \frac{P}{k_1} \left\{ \log[P] - \log \left[P e^{-k_1(\lambda - \psi)} - \frac{\gamma}{p^\delta} k_0 e^{-k_1 \lambda} + \frac{\gamma}{p^\delta} k_0 \right] \right\} \\
\Rightarrow \log[P] - \log \left[P e^{-k_1(\lambda - \psi)} - \frac{\gamma}{p^\delta} k_0 e^{-k_1 \lambda} + \frac{\gamma}{p^\delta} k_0 \right] &= 0 \\
\Rightarrow \log[P] &= \log \left[P e^{-k_1(\lambda - \psi)} - \frac{\gamma}{p^\delta} k_0 e^{-k_1 \lambda} + \frac{\gamma}{p^\delta} k_0 \right] \\
\Rightarrow P &= P e^{-k_1(\lambda - \psi)} - \frac{\gamma}{p^\delta} k_0 e^{-k_1 \lambda} + \frac{\gamma}{p^\delta} k_0 \\
\Rightarrow \frac{P + \frac{\gamma}{p^\delta} k_0 e^{-k_1 \lambda} - \frac{\gamma}{p^\delta} k_0}{P e^{-k_1 \lambda}} &= e^{k_1 \psi} \\
\Rightarrow \psi &= \frac{1}{k_1} \left\{ \log \left[e^{k_1 \lambda} \left(P - \frac{\gamma}{p^\delta} k_0 \right) + \frac{\gamma}{p^\delta} k_0 \right] - \log[P] \right\} \tag{12}
\end{aligned}$$

Total production in complete cycle is

$$\begin{aligned}
q &= P\beta + P(T + \lambda - T - \psi) \\
\Rightarrow q &= P\beta + P(\lambda - \psi) \\
\Rightarrow q &= P\{\beta + (\lambda - \psi)\}
\end{aligned}$$

Cost Components

$$\text{Set up cost} = \xi$$

$$\text{Production cost} = \left(C_p + \frac{C_0}{n^\phi} \right) P \{ \beta + (\lambda - \psi) \}$$

$$\begin{aligned}
\text{Holding cost} &= h \int_0^\beta Q(t).dt + h \int_\beta^T Q(t).dt \\
&= h \left[\int_0^\beta \left\{ (\alpha - 1) \frac{\gamma}{p^\delta} \left(t + \frac{\theta t^3}{6} \right) \text{Exp} \left[-\frac{\theta t^2}{2} \right] dt \right\} \right. \\
&\quad \left. + \int_\beta^T \left\{ \frac{\gamma}{p^\delta} \left[(T - t) + \frac{\theta}{6} (T^3 - t^3) \right] \text{Exp} \left[-\frac{\theta t^2}{2} \right] \right\} dt \right] \\
&= h(\alpha - 1) \frac{\gamma}{p^\delta} \left(\frac{\beta^2}{2} - \frac{\theta \beta^4}{12} \right) \\
&\quad + \frac{h\gamma}{p^\delta} \left\{ \frac{T^2}{2} + \frac{\theta T^4}{12} - \left(T\beta - \frac{\beta^2}{2} \right) - \frac{\theta}{6} \left(T^3\beta - \frac{\beta^4}{4} \right) + \frac{\theta}{2} \left(\frac{T\beta^3}{3} - \frac{\beta^4}{4} \right) \right\}
\end{aligned}$$

$$\text{Sales revenue} = p \left\{ \frac{\gamma}{p^\delta} T + P(\lambda - \psi) \right\}$$

$$\text{Total profit} = \text{Sales revenue} - \text{set up cost} - \text{manufacturing cost} - \text{holding cost}$$

Total profit

$$\begin{aligned}
&= \left[p \left\{ \frac{\gamma}{p^\delta} T + P(\lambda - \psi) \right\} - \left(C_p + \frac{C_0}{n\phi} \right) P \{ \beta + (\lambda - \psi) \} - \xi - h(\alpha - 1) \right. \\
&\quad \left. \frac{\gamma}{p^\delta} \left(\frac{\beta^2}{2} - \frac{\theta \beta^4}{12} \right) - \frac{h\gamma}{p^\delta} \left\{ \frac{T^2}{2} + \frac{\theta T^4}{12} - \left(T\beta - \frac{\beta^2}{2} \right) - \frac{\theta}{6} \left(T^3\beta - \frac{\beta^4}{4} \right) + \frac{\theta}{2} \left(\frac{T\beta^3}{3} - \frac{\beta^4}{4} \right) \right\} \right]
\end{aligned}$$

(13)

Average Profit (TP)

$$= \frac{1}{T + \lambda} \left[p \left\{ \frac{\gamma}{p^\delta} T + P(\lambda - \psi) \right\} - \left(C_p + \frac{C_0}{n^\phi} \right) P\{\beta + (\lambda - \psi)\} - \xi - h(\alpha - 1) \right. \\ \left. \frac{\gamma}{p^\delta} \left(\frac{\beta^2}{2} - \frac{\theta\beta^4}{12} \right) - \frac{h\gamma}{p^\delta} \left\{ \frac{T^2}{2} + \frac{\theta T^4}{12} - \left(T\beta - \frac{\beta^2}{2} \right) - \frac{\theta}{6} \left(T^3\beta - \frac{\beta^4}{4} \right) + \frac{\theta}{2} \left(\frac{T\beta^3}{3} - \frac{\beta^4}{4} \right) \right\} \right]$$

(14)

Now we have to maximize the average Profit

Such that $T \geq 0$ and $\lambda \geq 0$

4. Numerical Example

Example: The above theoretical results are verified through a numerical example; we have considered the following input parameters in appropriate units.

$\alpha = 1.5, C_p = 10, \phi = 0.45, P = 100, n = 5, \delta = 1.25, C_0 = 2, \theta = 0.01, h = 1.2, k_0 = 0.85,$
 $k_1 = 0.45, p = 15, \gamma = 12,00,000, \lambda = 1.75, \xi = 750$

Putting the values of given parameters and solve the problem, the optimal replenishment policy for the problem is $T = 1.245, \beta = 0.6220, \psi = 0.4862$ and the average profit is 5248.50.

5. Sensitivity Analysis

Table 1. Optimal results for the same set of values as in the given example for deterioration rate θ

S.No.	θ	T	β	ψ	TP
1.	.006	1.1274	0.6620	0.2562	5795.32
2.	.008	1.1882	0.6562	0.3589	5462.27
3.	.010	1.2450	0.6220	0.4862	5248.50
4.	.012	1.3320	0.6161	0.5124	4962.58
5.	.014	1.4125	0.6173	0.6214	4478.06

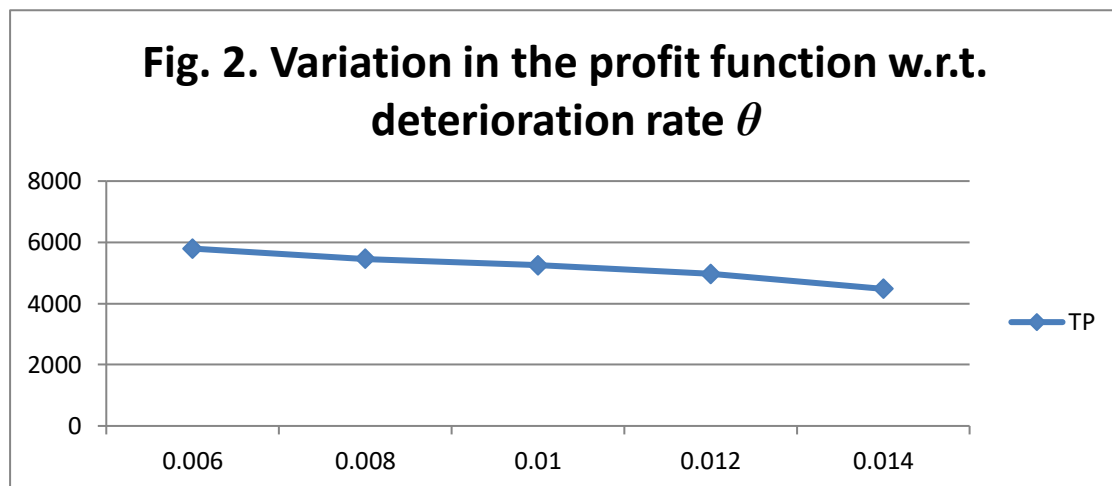


Table 2. Optimal results for same set of values as in given example for holding cost h

S.No.	h	T	β	ψ	TP
1.	1.0	1.1274	0.6238	0.4625	5935.52
2.	1.1	1.1882	0.6265	0.4785	5652.78
3.	1.2	1.2450	0.6270	0.4862	5248.50
4.	1.3	1.3320	0.6321	0.4921	4763.25
5.	1.4	1.4125	0.6335	0.5123	4325.57

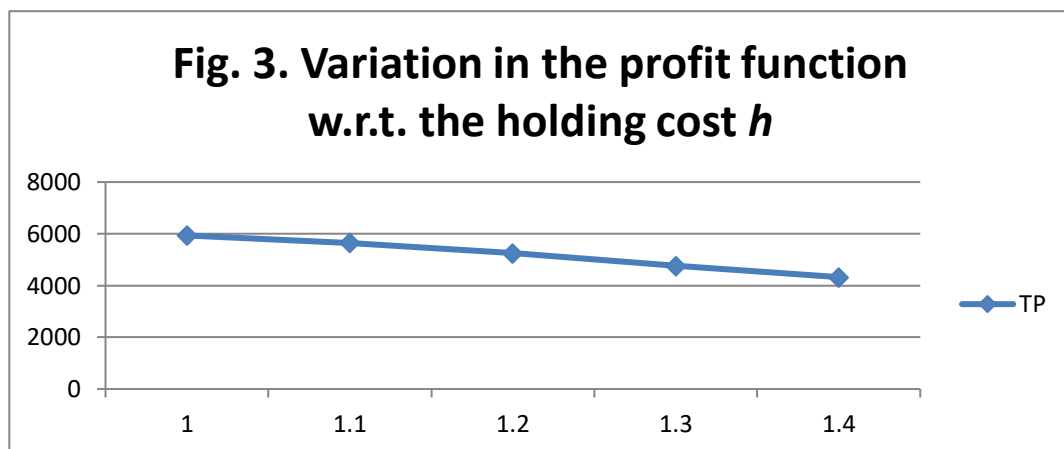


Table 3. Optimal results for the same set of values as in the given example for production cost C_p

S.No.	C_p	T	β	ψ	TP
1.	8	1.1556	0.6138	0.4520	6245.63
2.	9	1.1922	0.6185	0.4635	5925.34
3.	10	1.2450	0.6270	0.4862	5248.50
4.	11	1.2752	0.6322	0.48969	4935.80
5.	12	1.3511	0.6451	0.5021	4635.82

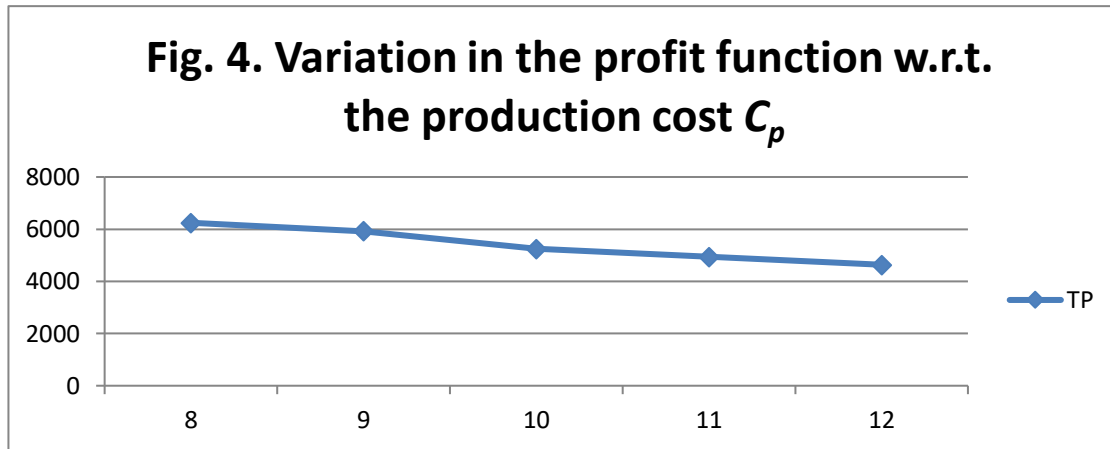


Table 4. Optimal results for the same set of values as in the given example for demand parameter γ

S.No.	γ	T	β	ψ	TP
1.	10,00,000	1.2162	0.6965	0.5263	4526.58
2.	11,00,000	1.2296	0.6542	0.5017	4863.48
3.	12,00,000	1.2450	0.6270	0.4862	5248.50
4.	13,00,000	1.2691	0.5926	0.4512	5638.49
5.	14,00,000	1.2843	0.5413	0.4382	5946.19

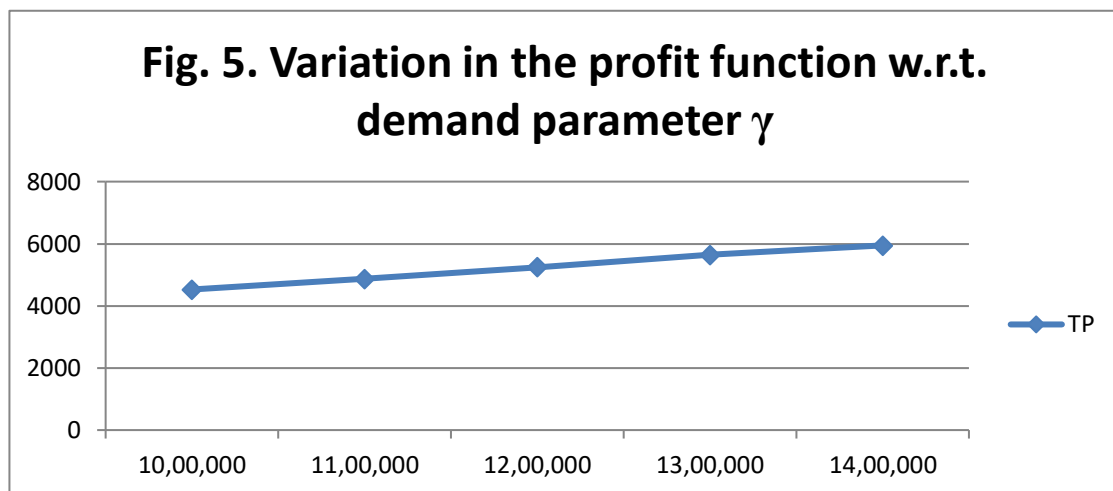
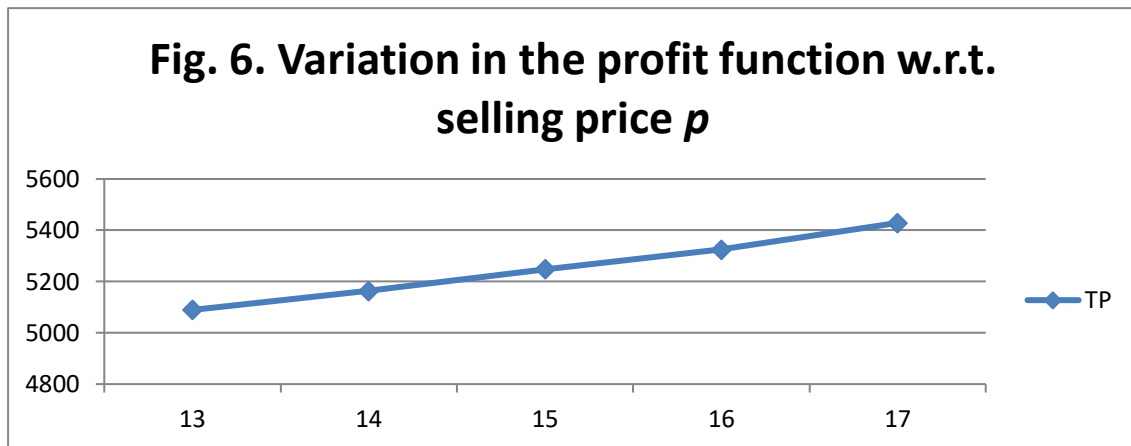


Table 5. Optimal results for the same set of values as in given example for selling price p

S.No.	p	T	β	ψ	TP
1.	13	1.2262	0.6153	0.4563	5089.35
2.	14	1.2359	0.6199	0.4732	5163.05
3.	15	1.2450	0.6270	0.4862	5248.50
4.	16	1.2548	0.6315	0.4964	5324.19
5.	17	1.2694	0.6385	0.5084	5428.72

**Observations**

1. From the table 1, it is observed that as the deterioration rate increases, time T and ψ increases while β and TP decreases.

2. While table 2, Reveals that the increment in holding cost resulted in the reduction in the profit TP and T, ψ and β increases.

3. From the table 3, With the increment in production cost T, β and ψ increases while TP decrease.

4. From the table 4, With the increment in demand parameter T and TP, increases while β and ψ decrease.

5. From the table 5, With the increment in price T, ψ , β and TP increases.

6. Conclusions

Here in this study, our economic production lot sizing model deals with the realistic assumptions, such as perishable items, stock out situation, learning effect. Deteriorating rate is taken as time-dependent which is much better than the constant deterioration which is taken by most of the researchers. Demand is a function of a selling price. As it is well known that demand is always affected by the price, usually it seems to us that higher the price lowers the demand rate. Shortages are allowed and the partially backlogged. The backlogging rate is considered to be waiting time dependent. It is the exponential function of waiting time. Our model can be so much beneficial for the practitioners because of its flexibility. This model helps the policymakers to control the inventory.

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