International Transactions in Mathematical Sciences and Computers
Peer Reviewed and Refereed Journal
Jan-June 2025, Volume 18 No. 1 pp.1-18
ISSN-(Print) 0974-5068, (Online) 0975-3753

https://doi.org/10.58517/ITMSC.2025.18101

Impact of inflation on sustainable inventory model with stock dependent demand with carbon emission

A.B. Chandramauli, *Santosh , Shrestha Pundir

Department of Mathematics, Meerut College Meerut, India
Thapar Institute of Engineering and Technology, Deemed to be University, Patiala, India
Correspondence mail: santosh.yyadav@gmail.com

Abstract: In the wake of environmental concerns and the growing recognition of climate change's impacts, the integration of sustainability principles into inventory management practices has become imperative. This study proposes a novel sustainable inventory model tailored for green warehouse farms, aiming to minimize carbon emissions while effectively managing inventory amidst inflationary pressures and stock-dependent demand fluctuations. The model incorporates controllable carbon emissions as a key performance indicator, aligning with the sustainability objectives of green warehouse farms. Recognizing the dual objectives of minimizing emissions and meeting customer demand, the model optimizes inventory levels while considering the carbon footprint associated with various inventory management decisions. In this study non-instantaneous deterioration to show the lifetime of product freshness when presence of a constant backorder. The objective of this model is to minimise total cost. The proposed model is validated through a numerical example. Conducting sensitivity analysis allows us to investigate the impact of different parameters on the optimal solution.

keywords: Sustainable inventory management, carbon emissions, green warehouse farms, inflation, stock-dependent demand, optimization.

1. Introduction and Literature Review

In the face of increasing environmental concerns and the imperative for sustainable business practices, the integration of environmentally conscious strategies into supply chain management has become paramount. Within this context,

*Corresponding Author

the development of a sustainable inventory model tailored to green warehouse farms presents a significant opportunity to mitigate carbon emissions while optimizing operational efficiency. This model aims to address several critical factors, including controllable carbon emissions, inflation, stock-dependent demand, non-instantaneous deterioration, and supplier request prepayment in multiple installments. Green warehouse farms represent a nexus of agricultural production and environmentally friendly practices, striving to minimize ecological impact while meeting the demands of a growing population. However, traditional inventory management approaches often neglect environmental considerations and fail to account for dynamic factors such as carbon emissions, inflation, and varying demand patterns. The proposed sustainable inventory model acknowledges these complexities and seeks to provide a comprehensive framework for decision-making within green warehouse farms. By integrating controllable carbon emissions tracking and reduction strategies into inventory management, the model aims to optimize resource utilization while minimizing environmental footprint. This involves careful consideration of sourcing practices, transportation logistics, and energy consumption within the warehouse facility. In this manner we discuss the development of the traditional economic order quantity inventory model. Deterioration affects inventory management for perishable products like vegetables, fruits, flowers, gasoline, and chemicals (Nahmias, 1982). By definition, deterioration is the process which reduces quality and quantity of the product. The effects of this deterioration rate have been investigated extensively in the literature. Chen *et al.* (2013) developed a model for reduce carbon emission costs and modify order quantities, and then used their model to control carbon emission for a variety of environmental regulations. Zhang *et al.* (2014) developed an economic production model in which they considered advance prepayment scheme and no relationship in payment length. Taleizadeh (2014) Created a deteriorating EOQ model for a purchasing inventory system with multiple prepayments. He talked about EOQ models with and without shortages for prepayments. Shorter prepayment periods lead to higher system costs. Many shops use a two-warehouse policy to manage their inventories. The retailer has an owned warehouse (OW) with a specific capacity. If the number of items exceeds this limit, a rented warehouse (RW) is used to keep the excess items safe. Typically, the buyer is served from the RW first, followed by the OW. Using these concepts, Shah and Cardenas-Barron (2015) developed an inventory model in which the retailer's credit policy and ordering decisions were examined after the supplier offered the retailer a cash discount or a set credit period. Jaggi et al. (2015) resolved a deteriorating inventory model with imperfect items using a two-warehouse policy. They implemented a new inventory system with a two-warehouse policy for deteriorating and defective products. Lou et al. (2015) developed a model with linear demand. Teng et al. (2016) investigated deteriorating products with dates of expiration and demonstrated that their model was closer to reality than other models. Tiwari et al. (2016) used a trade credit policy and inflation to determine optimal replenishment policies for non-instantaneously deteriorating products during shortages. They analysed multiple numerical examples to maximize cost. Majumder et al. (2016) presented an integrated inventory model with a two-level credit period. Xu et al. (2017) investigated a two-warehouse inventory system with a finite time horizon, comparing various dispatching systems for declining products, and demonstrated the uniqueness and existence of optimal solutions. Wu et al. (2017) Assumed advance-cash-or-credit methods of payment and incorporated certain key facts to determine the optimal fraction of cycle time for prepayments in order to maximize total cost. Datta (2017) developed economic production model with linear and time dependent demand. Chakraborty et al. (2018) expanded Jaggi et al.'s (2015) work, which allowed for three-parameter Weibull distribution deterioration and ramp-type demand in an inflationary environment with payment delays. Mishra et al. (2019) studied the impact of price and stock-dependent demand on a deteriorating inventory model with trade credit policy. They developed a method for determining cycle time, maximum price, and cost. Khalilpourazari et al. (2019) investigate a model where demand depend upon time and selling price. Mishra et al. (2020) created inventory model for non-instantaneous deterioration and use technology. Chandra (2020) investigated inventory model for deteriorating

products with varying demand types, but no one was concerned about the system's environmental impact, and no one advocated for the adoption of green technology and also consider the concept of two-warehouses. Wee and Daryanto (2020) investigated inventory models with a carbon emission tax system that accommodated backorders which is low-carbon supply chain. Mishra et al. (2020b) developed a sustainable inventory model that includes controllable emissions and can account for deterioration. The study highlighted the advantages of green technology and proposed a retailer-friendly greenhouse model with a single warehouse. This suggests that two-warehouse inventory systems are crucial for reducing emissions through green technology.

2. Notations and Assumptions

This model has following assumptions:

1. Infinite planning horizon for the system.
2. Lead time of this inventory system is taken as zero.
3. Shortage is allowed with partial backlogged at rate η.
4. Non-instantaneous deterioration is considered. At OW deterioration rate is α and at RW deterioration rate is β.
5. The provider asks prepayment in n installments. The store obtains a loan from a financial institution with a specific interest rate. Throughout this analysis, a comparable technique is utilized to calculate the cyclic capital profit of prepayments, as described in Taleizadeh et al. (2013) and Lashgari et al. 2018.
6. Inflation is allowed.
7. The fraction of the emission reduction due to advanced technology G is $F = \zeta(1 - e^{\chi G})$.

Table:1- shown notation which use in this model

Constant parameters	**Explanation**
a	Constant
b	Constant
c	Constant
p	Selling price
$I_r(t)$	Inventory level for rented warehouse
$I_o(t)$	Inventory level for own warehouse
S_r	Capacity of own warehouse
β	Deterioration rate
α	Deterioration rate
η	Backorder rate

R	Backorder quantity
ξ	Inflation rate
ζ	Amount of carbon emission
c_p	Purchasing cost
C_{h_r}	Holding profit for rented warehouse
C_{h_o}	Holding profit for own warehouse
c_s	Backorder profit per unit
O_r	Ordering profit
I_p	Interest payment
σ	Fraction of purchasing cost
n	Number of instalments
N	Trip number
L	Total lead time for delivery of the product
Q	Order quantity
c_1	Minimum transportation profit needed to transport products
c_2	Fuel consumption when the truck is empty
d	Distance travelled from own warehouse to rented warehouse and from rented warehouse to customer
c_3	Supplementary fuel consumption of the truck per ton of payload
m	Product weight
e_1	Carbon emission profit produced by the vehicle
e_2	Extra carbon emission profit for transporting one unit of an item
G	Green technology investment

3. Mathematical Formulation

Here a model is proposed which is two-green-warehouse inventory model. A supplier requests that retailer submit a portion of the purchasing profit which is σ before the transport of the products and paid in n equal instalments through equal interval of lead time L. when the product received (t=0) the retailer is asked to submit the remaining part $1-\sigma$ of the purchasing cost. Retailer keeps some demanded items in own warehouse and other in rented warehouse transported by a truck. So, there are some costs in this transportation fixed and variable both, carbon emission profit also. Green technology helps reduce CO2 emissions, which are harmful to the environment. The technology is used precisely to benefit retailers. When receiving the lot, R units of product are used to satisfy the partial backlogged demand, and the on-hand inventory level afterward drops to S. S_r units of purchasable products are reserved in the OW, while the residual portion of the purchasable units ($S-S_r$) is kept in the RW.

The RW has a higher holding profit than the OW due to its ownership by a third party and potential for better service. In the interval (0, t_1) the inventory level is decreases in RW due to demand $D = (a - bp) + cI_r(t)$and deterioration rate β.

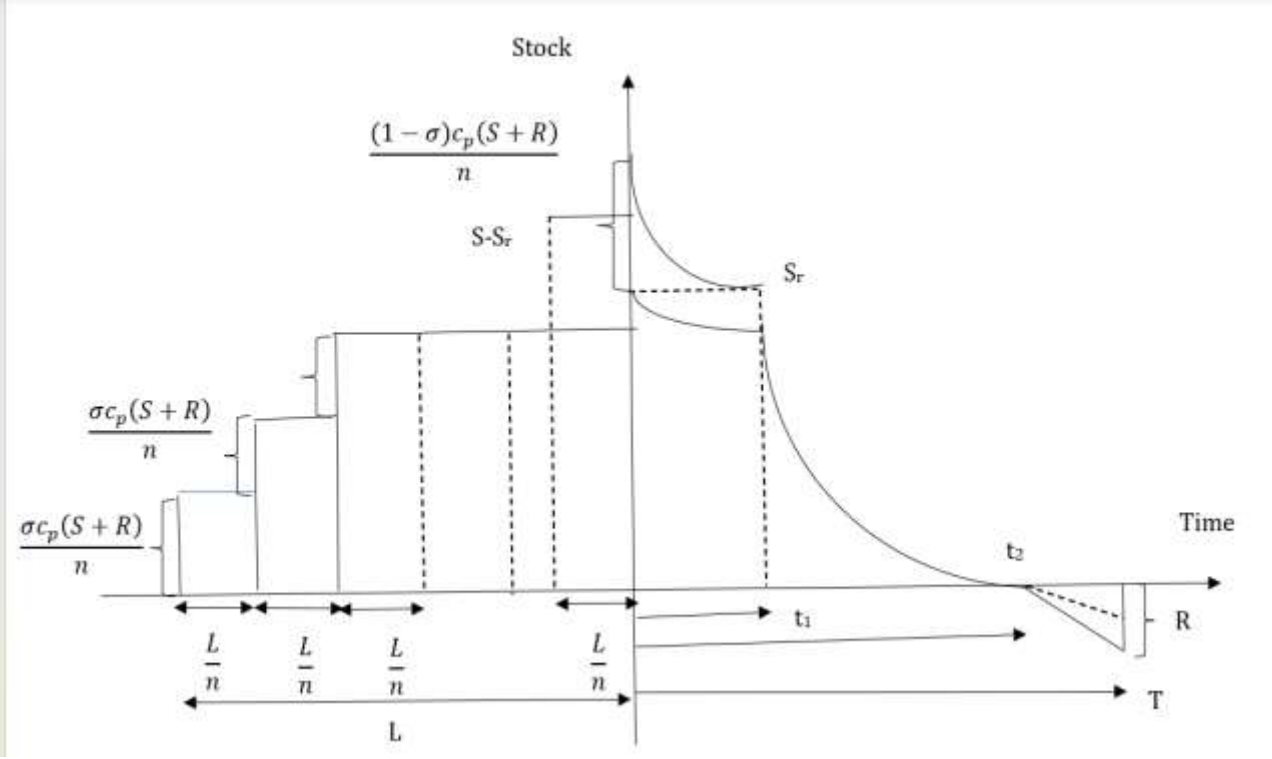

Fig.1: Advance payment and two warehouse inventory system

At any time, t the inventory level $I_r(t)$in the RW is defined by the resulting differential equations (DEs):

$$\frac{dI_r(t)}{dt} + \beta I_r(t) = -D + cI_r(t), 0 \le t \le t_1$$

$$\frac{dI_r(t)}{dt} + (\beta - c)I_r(t) = -D \quad (1)$$

$$I_r(0) = S - S_r, I_r(t_1) = 0 \quad (2)$$

$$I_r(t) = \frac{D}{\beta - c}\left(e^{(\beta-c)(t_1-t)} - 1\right) \quad (3)$$

$$S = S_r + \frac{D}{\beta - c}\left(e^{(\beta-c)t_1} - 1\right) \quad (4)$$

Again, the level of inventory denoted by $I_o(t)$ in OW at any time t, differential equation are written below:

$$\frac{dI_o(t)}{dt} + \alpha I_o(t) = 0, 0 \le t \le t_1 \quad (5)$$

$$\frac{dI_o(t)}{dt} + \alpha I_o(t) = -D + cI_o(t), t_1 \le t \le t_2 \quad (6)$$

$$\frac{dI_o(t)}{dt} = -\mu D, t_2 \le t \le T \quad (7)$$

$$with\ I_o(t) = S_r, I_o(t_2) = 0, I_o(T) = -R \quad (8)$$

After solving equation (5), (6) and (7), we get

$$I_o(t) = S_r e^{-\alpha t}, 0 \le t \le t_1 \quad (9)$$

$$I_o(t) = \frac{D}{\alpha - c}\left(e^{(\alpha - c)(t_2 - t)} - 1\right), t_1 \le t \le t_2 \quad (10)$$

$$I_o(t) = \eta D(T - t) - R, t_2 \le t \le T \quad (11)$$

$$R = \eta D(T - t_2) \quad (12)$$

Hence, total order quantity is:

$$Q = S + R = S_r + \frac{D}{\beta - c}\left(e^{(\beta - c)t_1} - 1\right) + \eta D(T - t_2) \quad (13)$$

Ordering cost= O_r (14)

Purchasing cost=$c_p(S + R)$

$$= c_p\left[S_r + \frac{D}{\beta - c}\left(e^{(\beta - c)t_1} - 1\right) + \eta D(T - t_2)\right] \quad (15)$$

Holding cost= $C_{h_r}\int_0^{t_1} e^{-\xi t} I_r(t)dt + C_{h_o}\int_0^{t_1} I_o(t)\, e^{-\xi t} + C_{h_o}\int_{t_1}^{t_2} e^{-\xi t}\, I_o(t)\, dt$

$$= \frac{C_{h_r} D}{\beta - c}\left[e^{-\xi t_1}\left(\frac{1}{c - \beta - \xi} + \frac{1}{\xi}\right) - \frac{e^{t_1(\beta - c)}}{c - \beta - \xi} - \frac{1}{\xi}\right] + C_{h_o} S_r\left[\frac{1}{\xi + \alpha}\left(1 - e^{-(\xi + \alpha)t_1}\right)\right] + \frac{C_{h_o} D}{\alpha - c}\left[\frac{e^{-\xi t_2}}{c - \xi - \alpha} + \frac{e^{-\xi t_2}}{\xi} - \frac{e^{-\xi t_1 + (\alpha - c)(t_2 - t_1)}}{c - \xi - \alpha} - \frac{e^{-\xi t_1}}{\xi}\right] \quad (16)$$

Sales revenue= $p[Dt_2 + \eta(T - t_2)]$

Shortage cost=$-C_s\int_{t_2}^{T} e^{-\xi t} I_o(t)dt$

$$= C_s\left[\frac{\eta DT}{\xi}\left(e^{-\xi T} - e^{-\xi t_2}\right) + \eta D\left(\frac{t_3 e^{(-\xi t_2)}}{\xi} - \frac{Te^{-\xi T}}{\xi} - \frac{e^{-\xi T}}{\xi} + \frac{e^{-\xi t_2}}{\xi}\right) + \frac{R}{\xi}\left(e^{-\xi t_2} - e^{-\xi T}\right)\right] \quad (17)$$

Lost sale cost=$C_i \int_{t_2}^{T}(1-\eta)Ddt$

$= C_i(1-\eta)D(T-t_2)$ (18)

Cyclic capital cost:

The cycle capital profitis calculated using the same procedure as for the purchaser formerly receiving the order in Taleizadeh et al. (2013), Teng et al. (2016), Wu et al. (2017), Lashgari et al. (2018).

$$CCC = \left(\frac{I_p\sigma c_p}{n}Q \times n \times \frac{L}{n}\right) + \left(\frac{I_p\sigma c_p}{n}Q \times (n-1) \times \frac{L}{n}\right) + \left(\frac{I_p\sigma c_p}{n}Q \times \left(n - n(n-2)\right) \times \frac{L}{n}\right) + \left(\frac{I_p\sigma c_p}{n}Q \times \left(n - (n-1)\right) \times \frac{L}{n}\right) = \left(\frac{I_p\sigma c_p}{n}Q \times \frac{L}{n}\right)[n + (n-1) + \cdots..+2+1]$$

$$= \frac{n+1}{2n}I_pL\sigma c_pQ \quad (19)$$

Transportation cost- The transportation profit comprises a fixed and variable transport profit together with a carbon emission cost. Here, the emission profit things on the delivery quantity (Q). the emission profit variable is calculated based on the weight of the shipment and relates to the size of the shipment. In this proposed model, the shipment size Q is transported or shipped from the OW to the RW and from the RW to customer, and the transport/shipping profit paid by retailer. Here, 2d is added, as the distance is also counted for the return trip (back and forth).

$$TNC = \frac{\mathrm{N}}{T}[c_1 + (2dt_vc_2 + dt_vc_3mQ) + 2de_1 + de_2Q)] \quad (20)$$

Green technology investment profit (GTI)=GT (21)

When implementation of green technology, then the new/ reduced transportation profit

$$TCR = \frac{\mathrm{N}}{T}\left[c_1 + (2dt_vc_2 + dt_vc_3mQ) + (2de_1 + de_2Q) \times \left(1 - \zeta(1 - e^{-\chi G})\right)\right] \quad (22)$$

The total profits

$$= p[Dt_2 + \eta(T-t_2)] - \left\{O_r + c_p\left[S_r + \frac{D}{\beta-c}\left(e^{(\beta-c)t_1} - 1\right) + \eta D(T-t_2)\right] + \frac{C_{h_r}D}{\beta-c}\left[e^{-\xi t_1}\left(\frac{1}{c-\beta-\xi} + \frac{1}{\xi}\right) - \frac{e^{t_1(\beta-c)}}{c-\beta-\xi} - \frac{1}{\xi}\right] + C_{h_o}S_r\left[\frac{1}{\xi+\alpha}\left(1 - e^{-(\xi+\alpha)t_1}\right)\right] + \frac{C_{h_o}D}{\alpha-c}\left[\frac{e^{-\xi t_2}}{c-\xi-\alpha} + \frac{e^{-\xi t_2}}{\xi} - \frac{e^{-\xi t_1+(\alpha-c)(t_2-t_1)}}{c-\xi-\alpha} - \frac{e^{-\xi t_1}}{\xi}\right] + C_s\left[\frac{\eta DT}{\xi}\left(e^{-\xi T} - e^{-\xi t_2}\right) + \eta D\left(\frac{t_2e^{(-\xi t_2)}}{\xi} - \frac{Te^{-\xi T}}{\xi} - \frac{e^{-\xi T}}{\xi} + \frac{e^{-\xi t_2}}{\xi}\right) + \frac{R}{\xi}\left(e^{-\xi t_2} - e^{-\xi T}\right)\right] + C_i(1-\eta)D(T-t_2) + \frac{n+1}{2n}I_pL\sigma c_pQ + \frac{\mathrm{N}}{T}\left[c_1 + (2dt_vc_2 + dt_vc_3mQ) + (2de_1 + de_2Q) \times \left(1 - \zeta(1 - e^{-\chi G})\right)\right] + GT\right\}$$

The total average profit:

$$= \frac{1}{T}\left[p[Dt_2 + \eta(T-t_2)] - \left\{O_r + c_p\left[S_r + \frac{D}{\beta-c}\left(e^{(\beta-c)t_1}-1\right) + \eta D(T-t_2)\right] + \frac{C_{h_r}D}{\beta-c}\left[e^{-\xi t_1}\left(\frac{1}{c-\beta-\xi}+\frac{1}{\xi}\right) - \frac{e^{t_1(\beta-c)}}{c-\beta-\xi} - \frac{1}{\xi}\right] + C_{h_o}S_r\left[\frac{1}{\xi+\alpha}\left(1-e^{-(\xi+\alpha)t_1}\right)\right] + \frac{C_{h_o}D}{\alpha-c}\left[\frac{e^{-\xi t_2}}{c-\xi-\alpha} + \frac{e^{-\xi t_2}}{\xi} - \frac{e^{-\xi t_1+(\alpha-c)(t_2-t_1)}}{c-\xi-\alpha} - \frac{e^{-\xi t_1}}{\xi}\right] + C_s\left[\frac{\eta DT}{\xi}\left(e^{-\xi T} - e^{-\xi t_2}\right) + \eta D\left(\frac{t_2 e^{(-\xi t_2)}}{\xi} - \frac{Te^{-\xi T}}{\xi} - \frac{e^{-\xi T}}{\xi} + \frac{e^{-\xi t_2}}{\xi}\right) + \frac{R}{\xi}\left(e^{-\xi t_2} - e^{-\xi T}\right)\right] + C_i(1-\eta)D(T-t_2) + \frac{n+1}{2n}I_p L\sigma c_p Q + \frac{\mathrm{N}}{T}\left[c_1 + (2dt_v c_2 + dt_v c_3 mQ) + (2de_1 + de_2 Q)\times\left(1-\zeta(1-e^{-\chi G})\right)\right] + GT\right\}\right] \quad (23)$$

1. **Solution methodology:** Here three independent variables so to optimize the total profit function the following steps are followed:
   1. Find first order derivative with respect to $t_1, T \; and \; G$ i.e,
   $$\frac{\partial TP}{\partial t_1}, \frac{\partial TP}{\partial T}, \frac{\partial TP}{\partial G}.$$
   2. Solve $\frac{\partial TP}{\partial t_1} = 0, \frac{\partial TP}{\partial T} = 0, \frac{\partial TP}{\partial G} = 0 \; and \; find \; t_1, T \; and \; G.$
   3. $\frac{\partial^2 TP}{\partial t_1^2}, \frac{\partial^2 TP}{\partial T^2}, \frac{\partial^2 TP}{\partial G^2}, \frac{\partial^2 TP}{\partial t_1 \partial T}, \frac{\partial^2 TP}{\partial T \partial G}, \frac{\partial^2 TP}{\partial t_1 \partial G}, \frac{\partial^2 TP}{\partial T \partial t_1}, \frac{\partial^2 TP}{\partial G \partial t_1}, \frac{\partial^2 TP}{\partial G \partial T}$ also find.
   4. Drive Hessian matrix W

$$W = \begin{bmatrix} \frac{\partial^2 TP}{\partial t_1^2} & \frac{\partial^2 TP}{\partial t_1 \partial T} & \frac{\partial^2 TP}{\partial t_1 \partial G} \\ \frac{\partial^2 TP}{\partial T \partial t_1} & \frac{\partial^2 TP}{\partial T^2} & \frac{\partial^2 TP}{\partial T \partial G} \\ \frac{\partial^2 TP}{\partial G \partial t_1} & \frac{\partial^2 TP}{\partial G \partial T} & \frac{\partial^2 TP}{\partial G^2} \end{bmatrix}$$

   5. Calculate all principal minor of matrix W ($i.e. W_{11}, W_{22}, W_{33}$).
   6. If all principal minors of this Hessian matrix W, are negative then the total profit is concave.

4. Numerical illustrations

Consider a business situation in which input parameters are given below

Parameter	Values	Parameters	Values	Parameters	Values
O_r	10 lakhs	p	10	η	0.001
C_p	5 Lakhs	c	0.1	χ	100
S_r	0.1 $	β	0.01	ζ	40
a	200	α	0.01	σ	0.01
b	10	ξ	1	n	0.005
L	0.05	Q	10	t_2	59
S_r	10	N	10	i_p	100
C_{ho}	0.1 $	C_{hr}	500 Rupees	C_s	0.02 $

C_i	20 Lakhs	d	3900 km	t_v	0.1
m	10	e_1	0.08	e_2	5
c_1	0.04	c_2	1	c_3	10
R	50				

Optimal result

Parameters	Values
t_1	33.7116 days
T	335.894 days
G	269.117 $
Total profit	2824.95 Lakhs

5. Concavity

For the global solution of total profit, we obtain the concavity graph which is shown in Figure. 2 Graphical representation of this inventory system is shown in below figures.

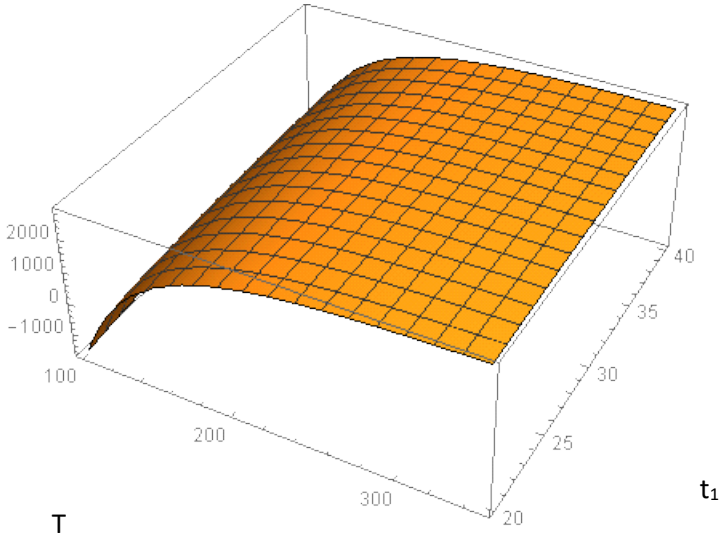

Fig:2- concavity shown between T and t_1.

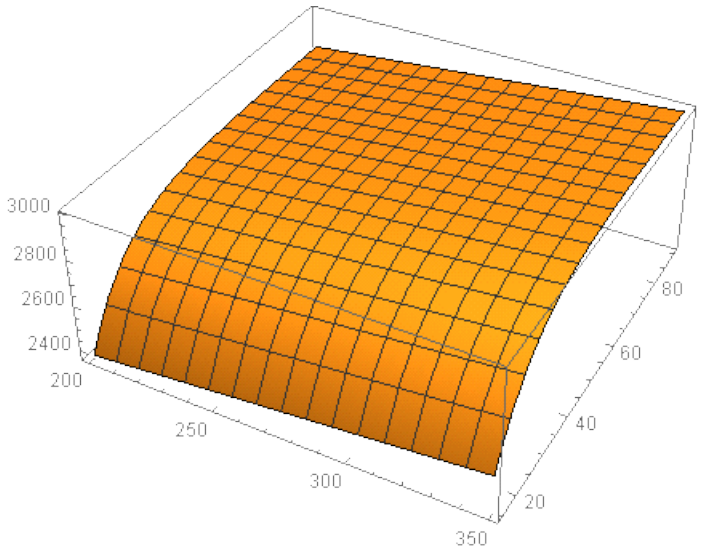

Fig:3- concavity shown between G and t_1

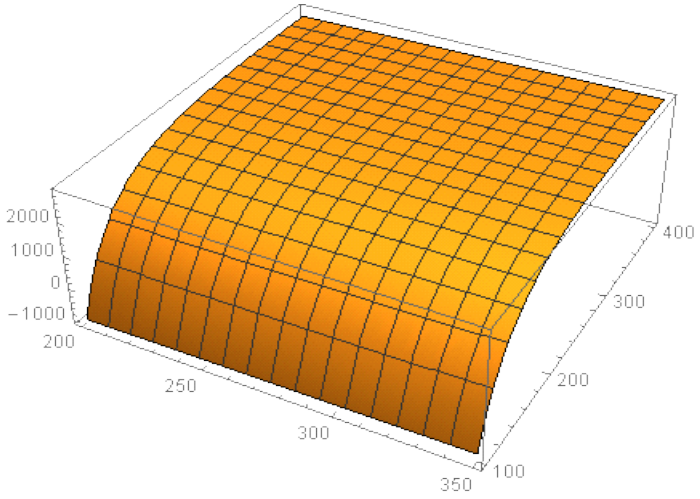

Fig:5- concavity shown between G and T

6. Sensitivity Analysis

Parameters	Percentage	t_1	T	G	Total cost
O_r	+25%	29.0847	330.827	260.987	2820.986
	+15%	30.892	334.987	240.98	3456.98
	-15%	33.7116	335.894	269.117	2824.95
	-25%	34.98476	330.637	279.958	3423.87
C_p	+25%	37.9832	334.9387	340.7368	4679.86
	+15%	34.9873	335.65	342.674	4073.37
	-15%	36.836	456.98	260.987	2347.847
	-25%	34.9837	345.76	240.98	3528.827
S_r	+25%	30.87	234.098	269.117	4829.264
	+15%	26.634	548.64	279.958	2820.986
	-15%	25.5163	349.087	340.7368	3456.98
	-25%	37.653	287.647	342.674	2824.95
a	+25%	26.873	567.93	260.987	3423.87
	+15%	29.765	345.743	240.98	4679.86
	-15%	23.8365	345.756	269.117	4073.37
	-25%	40.87	234.867	279.958	2347.847

b	+25%	58.643	330.827	340.7368	3528.827
	+15%	20.84	330.827	342.674	4829.264
	-15%	27.465	330.827	260.987	2820.986
	-25%	34.75	330.827	240.98	3456.98
p	+25%	23.645	234.098	269.117	2824.95
	+15%	22.74	548.64	279.958	3423.87
	-15%	41.345	349.087	340.7368	4679.86
	-25%	39.762	287.647	342.674	4073.37
c	+25%	34.867	567.93	260.987	2347.847
	+15%	19.34	345.743	240.98	3528.827
	-15%	29.0847	345.756	269.117	4829.264
	-25%	30.892	234.867	279.958	2820.986
β	+25%	33.7116	330.827	340.7368	3456.98
	+15%	34.98476	330.827	342.674	2824.95
	-15%	37.9832	330.827	260.987	3423.87
	-25%	34.9873	330.827	240.98	4679.86
α	+25%	36.836	234.098	269.117	4073.37
	+15%	34.9837	548.64	279.958	2347.847
	-15%	30.87	349.087	340.7368	3528.827
	-25%	26.634	287.647	342.674	4829.264
ξ	+25%	25.5163	567.93	260.987	2820.986
	+15%	37.653	345.743	240.98	3456.98
	-15%	26.873	345.756	269.117	2824.95
	-25%	29.765	234.867	279.958	3423.87
η	+25%	23.8365	330.827	340.7368	4679.86
	+15%	40.87	330.827	342.674	4073.37
	-15%	58.643	330.827	260.987	2347.847
	-25%	20.84	330.827	240.98	3528.827
χ	+25%	27.465	234.098	269.117	4829.264
	+15%	34.75	548.64	279.958	2820.986
	-15%	23.645	349.087	340.7368	3456.98
	-25%	22.74	287.647	342.674	2824.95
ζ	+25%	41.345	567.93	260.987	3423.87
	+15%	39.762	345.743	240.98	4679.86
	-15%	34.867	345.756	269.117	4073.37
	-25%	19.34	234.867	279.958	2347.847
σ	+25%	29.0847	330.827	340.7368	3528.827
	+15%	30.892	330.827	342.674	4829.264
	-15%	33.7116	330.827	260.987	2820.986
	-25%	34.98476	330.827	240.98	3456.98

n	+25%	37.9832	234.098	269.117	2824.95
	+15%	34.9873	548.64	279.958	3423.87
	-15%	36.836	349.087	340.7368	4679.86
	-25%	34.9837	287.647	342.674	4073.37
L	+25%	30.87	567.93	260.987	2347.847
	+15%	26.634	345.743	240.98	3528.827
	-15%	25.5163	345.756	269.117	4829.264
	-25%	37.653	234.867	279.958	2820.986
t_2	+25%	58.643	330.827	269.117	4073.37
	+15%	20.84	330.827	279.958	2347.847
	-15%	27.465	330.827	340.7368	3528.827
	-25%	34.75	330.827	342.674	4829.264
S_r	+25%	23.645	234.098	260.987	2820.986
	+15%	22.74	548.64	240.98	3456.98
	-15%	41.345	349.087	269.117	2824.95
	-25%	39.762	287.647	279.958	3423.87
N	+25%	34.867	567.93	340.7368	4679.86
	+15%	19.34	345.743	342.674	4073.37
	-15%	29.0847	345.756	260.987	2347.847
	-25%	30.892	234.867	240.98	3528.827
i_p	+25%	33.7116	330.827	269.117	4829.264
	+15%	34.98476	330.827	279.958	2820.986
	-15%	37.9832	330.827	340.7368	3456.98
	-25%	34.9873	330.827	342.674	2824.95
C_{ho}	+25%	36.836	234.098	260.987	3423.87
	+15%	34.9837	548.64	240.98	4679.86
	-15%	30.87	349.087	269.117	4073.37
	-25%	26.634	287.647	279.958	2347.847
C_{hr}	+25%	25.5163	567.93	340.7368	3528.827
	+15%	37.653	345.743	342.674	4829.264
	-15%	26.873	345.756	260.987	2820.986
	-25%	29.765	234.867	240.98	3456.98
C_s	+25%	23.8365	345.87	269.117	2824.95
	+15%	40.87	435.876	279.958	3423.87
	-15%	58.643	126.87	340.7368	4679.86
	-25%	20.84	437.98	342.674	4073.37
	+25%	27.465	330.827	260.987	2347.847

C_i	+15%	34.75	330.827	240.98	3528.827
	-15%	23.645	330.827	269.117	4829.264
	-25%	22.74	330.827	279.958	2820.986
d	+25%	41.345	234.098	340.7368	3456.98
	+15%	39.762	548.64	342.674	2824.95
	-15%	34.867	349.087	260.987	3423.87
	-25%	19.34	287.647	240.98	4679.86
t_v	+25%	29.0847	567.93	269.117	4073.37
	+15%	30.892	345.743	279.958	2347.847
	-15%	33.7116	345.756	340.7368	3528.827
	-25%	34.98476	234.867	342.674	4829.264
m	+25%	37.9832	330.827	260.987	2820.986
	+15%	34.9873	330.827	240.98	3456.98
	-15%	36.836	330.827	269.117	2824.95
	-25%	34.9837	330.827	279.958	3423.87
e_1	+25%	30.87	234.098	340.7368	4679.86
	+15%	26.634	548.64	342.674	4073.37
	-15%	25.5163	349.087	260.987	2347.847
	-25%	37.653	287.647	240.98	3528.827
e_2	+25%	26.873	567.93	269.117	4829.264
	+15%	29.765	345.743	279.958	2820.986
	-15%	23.8365	345.756	340.7368	3456.98
	-25%	40.87	234.867	342.674	2824.95
c_1	+25%	58.643	330.827	260.987	3423.87
	+15%	20.84	330.827	240.98	4679.86
	-15%	27.465	330.827	269.117	4073.37
	-25%	34.75	330.827	279.958	2347.847
c_2	+25%	23.645	234.098	340.7368	3528.827
	+15%	22.74	548.64	342.674	4829.264
	-15%	41.345	349.087	260.987	3423.87
	-25%	39.762	287.647	240.98	4679.86
c_3	+25%	34.867	567.93	269.117	4073.37
	+15%	19.34	345.743	279.958	2347.847
	-15%	23.645	345.756	340.7368	3528.827
	-25%	22.74	234.867	342.674	4829.264
R	+25%	41.345	330.827	279.958	4073.37
	+15%	39.762	330.827	340.7368	2347.847
	-15%	34.867	330.827	342.674	3528.827
	-25%	19.34	330.827	345.873	4829.264

7. Observation

From the sensitivity analysis following assumptions are made

1. On increasing in parameter O_r, $t_1$is decreasing. T is fluctuating. G is fluctuating. while total profit fluctuating.
2. On increasing in parameter C_p, t_1*is* increasing.T is increasing. G is decreasing, while total profit fluctuating.
3. On increasing in parameter S_r, t_1 T G while total profit increasing.
4. On increasing in parameter a, $t_1$is decreasing. T is increasing, G is fluctuating, while total profit increasing.
5. On increasing in parameter b, t_1 is increasing. T is decreasing, G is increasing, while total profit decreasing.
6. On increasing in parameter p, t_1 is increasing. T is decreasing, G is constant, while total profit fluctuating.
7. On increasing in parameter c , t_1 is increasing. T is decreasing, G is fluctuating, while total profit increasing.
8. On increasing in parameter β, $t_1$is increasing. T is decreasing, G is decreasing while total profit decreasing.
9. On increasing in parameter α, t_1 is increasing, T is increasing, G is decreasing while total profit decreasing.
10. On increasing in parameter ξ, $t_1$is increasing, T is decreasing, G is constant while total profit increasing.
11. On increasing in parameter η, t_1 is increasing, T is decreasing, G is fluctuating, while total profit increasing.
12. On increasing in parameter χ, t_1 is increasing, T is increasing, G is increasing, while total profits increasing.
13. On increasing in parameterζ, $t_1$is decreasing, T is increasing, G is decreasing, while total profit decreasing.
14. On increasing in parameter σ, t_1 is increasing, T is increasing, G is increasing while total profit decreasing.
15. On increasing in parameter$n$, t_1 is increasing, T is increasing, G is fluctuating, while total profit fluctuating.
16. On increasing in parameter L, t_1 T G while total profits fluctuating.

17. On increasing in parameter t_2, t_1 is increasing. T is decreasing, G is fluctuating, while total profit increasing.
18. On increasing in parameter S_r, t_1 is increasing. T is decreasing, G is fluctuating, while total profit increasing.
19. On increasing in parameter i_p, $t_1$is increasing, T is decreasing, G is constant while total profit increasing.
20. On increasing in parameter C_{ho}, t_1 is increasing. T is decreasing, G is fluctuating, while total profit increasing.
21. On increasing in parameter C_{hr}, $t_1$is increasing, T is decreasing, G is constant while total profit increasing.
22. On increasing in parameter C_s, t_1 is increasing. T is decreasing, G is fluctuating, while total profit increasing.
23. On increasing in parameter C_i,t_1 is increasing, T is increasing, G is increasing, while total profit increasing.
24. On increasing in parameter d, t_1 is increasing, T is increasing, G is increasing, while total profit increasing.
25. On increasing in parameter t_v, t_1 is increasing. T is decreasing, G is fluctuating, while total profit increasing.
26. On increasing in parameter m , t_1 is increasing, T is increasing, G is increasing, while total profit increasing.
27. On increasing in parameter e_1, t_1 is increasing, T is increasing, G is increasing, while total profit increasing.
28. On increasing in parameter c_1, t_1 is increasing. T is decreasing, G is fluctuating, while total profit increasing.
29. On increasing in parameter c_2, t_1 is increasing, T is increasing, G is increasing, while total profit increasing.
30. On increasing in parameter c_3, t_1 is increasing. T is decreasing, G is fluctuating, while total profit increasing.
31. On increasing in parameter R , t_1 is increasing, T is increasing, G is increasing, while total profit increasing.

8. Conclusion

The sustainable inventory model proposed for green warehouse farms offers a comprehensive framework for addressing the complexities of modern agricultural supply chains. By integrating strategies to track and reduce carbon emissions, manage inflation and stock-dependent demand, handle non-instantaneous deterioration, and structure supplier prepayment agreements, the model provides a robust

approach to fostering sustainability while ensuring operational efficiency and financial stability. Through its implementation, green warehouse farms can exemplify the principles of environmental stewardship, economic resilience, and social responsibility, driving positive change within the agricultural industry and contributing to a more sustainable future for our planet. This model can be extended with more ware houses, fuzzy can be applied. Model can be extended with smart production system. Through the implementation of the proposed model, green warehouse farms can navigate this journey with confidence, forging a path towards a brighter and more sustainable future for agriculture and the planet as a whole.

References

Chakraborty, D., Jana, D. K., & Roy, T. K. (2018). Two-warehouse partial backlogging inventory model with ramp type demand rate, three-parameter Weibull distribution deterioration under inflation and permissible delay in payments. *Computers & industrial engineering*, *123*, 157-179.

Chandra, S. (2020). Two warehouse inventory model for deteriorating items with stock dependent demand under permissible delay in payment. *J. Math. Comput. Sci.*, *10*(4), 1131-1149.

Chen, X., Benjaafar, S., & Elomri, A. (2013). The carbon-constrained EOQ. *Operations Research Letters*, *41*(2), 172-179.

Datta, T. K. (2017). Effect of green technology investment on a production-inventory system with carbon tax. *Advances in operations research*, *2017*.

Jaggi, C. K., Tiwari, S., & Shafi, A. (2015). Effect of deterioration on two-warehouse inventory model with imperfect quality. *Computers & Industrial Engineering*, *88*, 378-385.

Lashgari, M., Taleizadeh, A. A., & Sadjadi, S. J. (2018). Ordering policies for non-instantaneous deteriorating items under hybrid partial prepayment, partial trade credit and partial backordering. *Journal of the Operational Research Society*, *69*(8), 1167-1196.

Lou, G. X., Xia, H. Y., Zhang, J. Q., & Fan, T. J. (2015). Investment strategy of emission-reduction technology in a supply chain. *Sustainability*, *7*(8), 10684-10708.

Majumder, P., Bera, U. K., & Maiti, M. (2016). An EPQ model for two-warehouse in unremitting release pattern with two-level trade credit period concerning both supplier and retailer. *Applied Mathematics and Computation*, *274*, 430-458.

Mishra, U., Wu, J. Z., & Tseng, M. L. (2019). Effects of a hybrid-price-stock dependent demand on the optimal solutions of a deteriorating inventory system and trade credit policy on re-manufactured product. *Journal of Cleaner Production*, *241*, 118282.

Mishra, U., Wu, J. Z., Tsao, Y. C., & Tseng, M. L. (2020). Sustainable inventory system with controllable non-instantaneous deterioration and environmental emission rates. *Journal of Cleaner Production*, *244*, 118807.

Nahmias, S. (1982). Perishable inventory theory: A review. *Operations research*, *30*(4), 680-708.

Shah, N. H., & Cárdenas-Barrón, L. E. (2015). Retailer's decision for ordering and credit policies for deteriorating items when a supplier offers order-linked credit period or cash discount. *Applied Mathematics and Computation*, *259*, 569-578.

Srivastava, S. K. (2007). Green supply-chain management: a state-of-the-art literature review. *International journal of management reviews*, *9*(1), 53-80.

Taleizadeh, A. A. (2014). An economic order quantity model for deteriorating item in a purchasing system with multiple prepayments. *Applied Mathematical Modelling*, *38*(23), 5357-5366.

Taleizadeh, A. A., Pentico, D. W., Jabalameli, M. S., & Aryanezhad, M. (2013). An economic order quantity model with multiple partial prepayments and partial backordering. *Mathematical and Computer Modelling*, *57*(3-4), 311-323.

Teng, J. T., Cárdenas-Barrón, L. E., Chang, H. J., Wu, J., & Hu, Y. (2016). Inventory lot-size policies for deteriorating items with expiration dates and advance payments. *Applied Mathematical Modelling*, *40*(19-20), 8605-8616.

Tiwari, S., Cárdenas-Barrón, L. E., Khanna, A., & Jaggi, C. K. (2016). Impact of trade credit and inflation on retailer's ordering policies for non-instantaneous deteriorating items in a two-warehouse environment. *International Journal of Production Economics*, *176*, 154-169.

Wahab, M. I. M., Mamun, S. M. H., & Ongkunaruk, P. (2011). EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact. *International Journal of Production Economics*, *134*(1), 151-158.

Wee, H. M., & Daryanto, Y. (2020). Imperfect quality item inventory models considering carbon emissions. *Optimization and Inventory management*, 137-159.

Wu, J., Teng, J. T., & Chan, Y. L. (2018). Inventory policies for perishable products with expiration dates and advance-cash-credit payment schemes. *International Journal of Systems Science: Operations & Logistics*, *5*(4), 310-326.

Zhang, Q., Tsao, Y. C., & Chen, T. H. (2014). Economic order quantity under advance payment. *Applied Mathematical Modelling*, *38*(24), 5910-5921.

Zhu, W., & He, Y. (2017). Green product design in supply chains under competition. *European Journal of Operational Research*, *258*(1), 165-180.